Exploiting Resolution Proofs to Speed Up LTL Vacuity Detection for BMC

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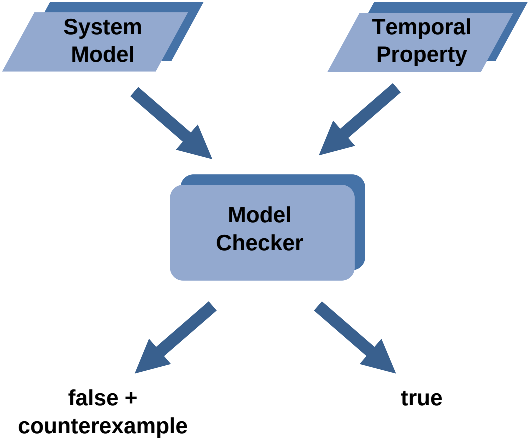
Nov 12, 2007 - FMCAD '07

Model Checking



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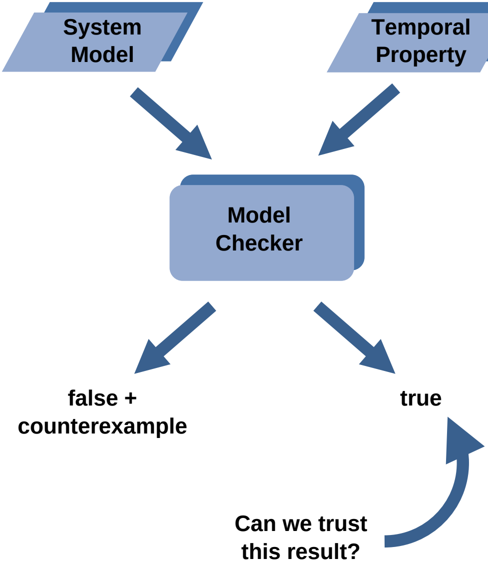

Sanity Checks

Errors in Model	**Errors in Environment**	**Errors in Property**
Debugging Overconstrained Declarative Models	Finding Environmental Guarantees	Vacuity Detection
[Shlyakhter et al. ’03]	[Chechik et al. ’07]	[Beer et al. ’99]
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GOAL: determine what parts of a property are not relevant

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- ...anything that can be substituted without changing the value of the property
- Example: "all requests are eventually serviced"
  LTL: $p = G(request \Rightarrow F\ serviced)$
  holds in a model that does not produce any requests!

Naive Vacuity Detection [Beer et al. '97, Kupferman and Vardi '99]

Test by substituting each subformula to check which ones are vacuous

EXAMPLE: "all requests are eventually serviced"

formalized as $p \; = \; G \text{ (request } \; \Rightarrow \; F \text{ serviced)}$

SOLUTION: four model checking runs

- $p_1 \; = \; G \text{ (true } \; \Rightarrow \; F \text{ serviced)}$
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Can be done without any special purpose tools

# of extra model checking runs grows with size of property

Brief Overview of Vacuity Detection

	Main Idea	Logic	Tool
[Beer et al. ’97]	Replace single occurrence of a subformula with true, false	w-ACTL	RuleBase
[Kupferman and Vardi ’99]	Generalized Beer’s definition	CTL*	–
[Armoni et al. ’03]	Introduced trace vacuity	LTL	Forecast Thunder
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Definition of vacuity used in this work [Gurfinkel and Chechik '04]

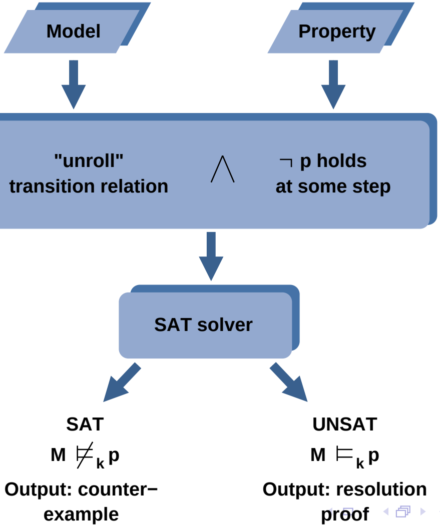
Property p is vacuous w.r.t. variable v iff $M \models p[v \leftarrow x]$, where x is an unconstrained model variable

None of these definitions target SAT-based BMC

Bounded Model Checking (BMC)

Check if property p holds up to k steps on model M: $M \models_k p$

- i.e., can we reach a state in k steps that satisfies $\neg$ p?



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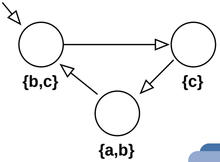
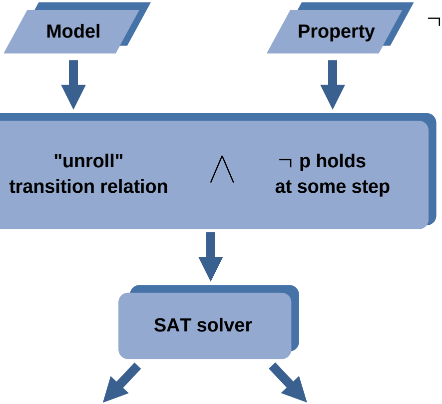


$\neg b \vee c$



k = 0

SAT

$M \not\models_k p$

Output: counter−example

UNSAT

$M \models_k p$

Output: resolution proof

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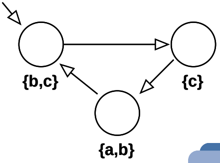

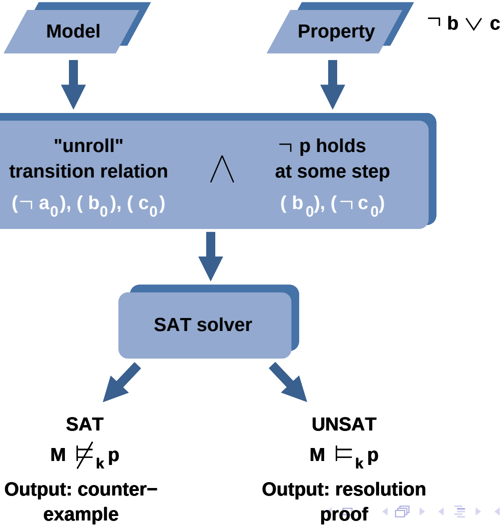

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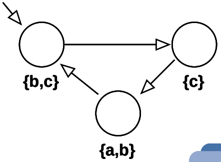

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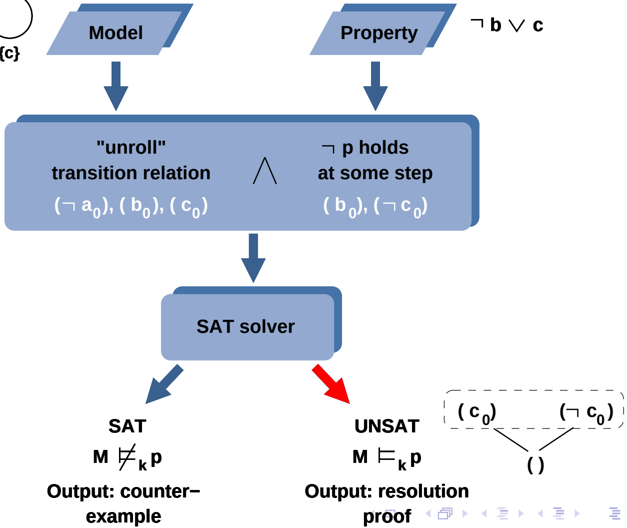

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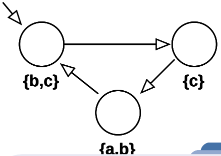
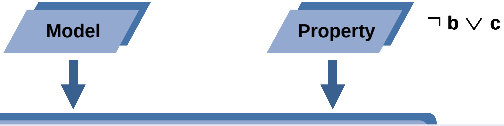
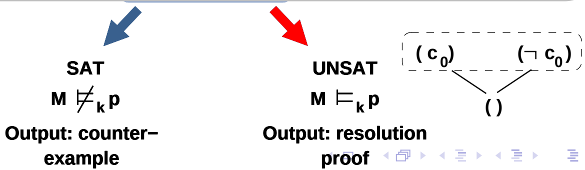




GOAL: use resolution proof for vacuity detection

- focus on variable vacuity
- use naive detection as baseline for comparison



Outline

- Model Checking
- Sanity Checks
- Naive Vacuity Detection
- Brief Overview of Vacuity Detection
- Bounded Model Checking
- New methods:
    - Irrelevance
    - Local Irrelevance
    - Peripherality
- Implementation: VaqTree
- Experiments
- Conclusions and Future Work

Algorithm 1 - Irrelevance

Model	$(\neg b \vee \neg c), (b), (\neg e), (d \vee f)$
Property	$(\neg a), (a \vee b), (\neg b \vee c), (d \vee e \vee f), (a \vee \neg c \vee d)$

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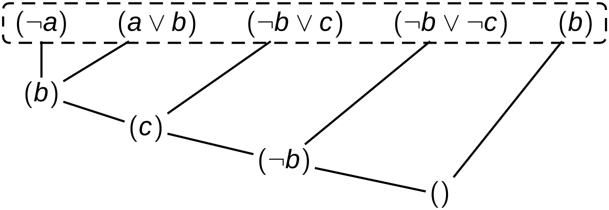

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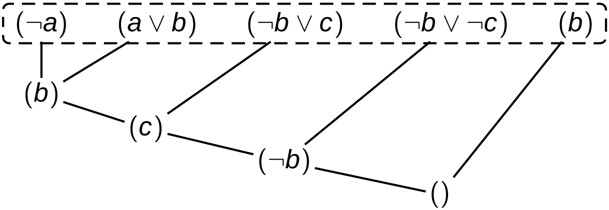

Variables in the property but not in the UNSAT core are irrelevant

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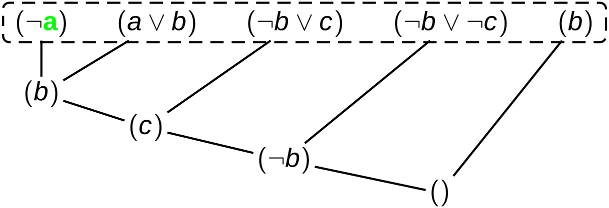

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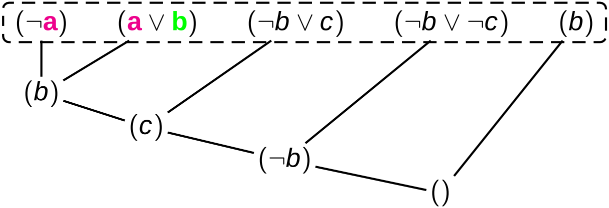

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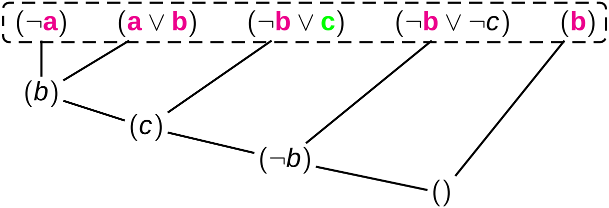

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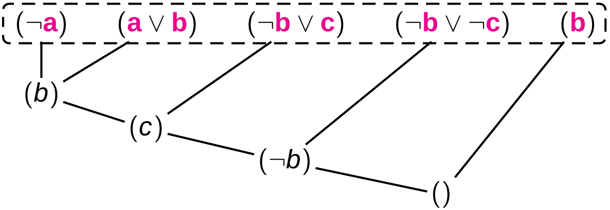

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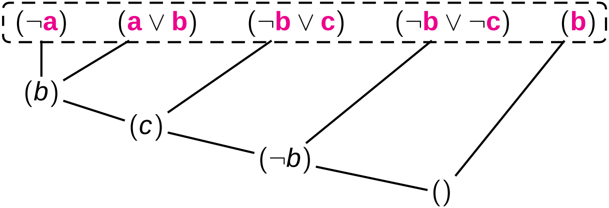

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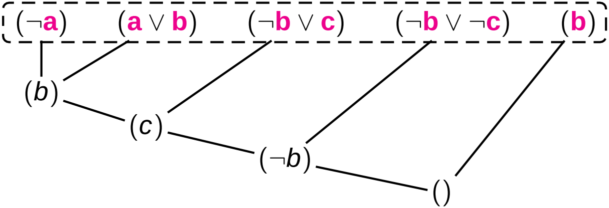

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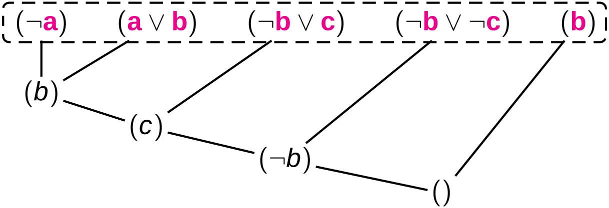

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- Linear in size of UNSAT core
- Very incomplete

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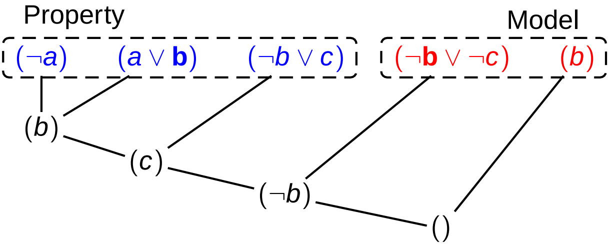

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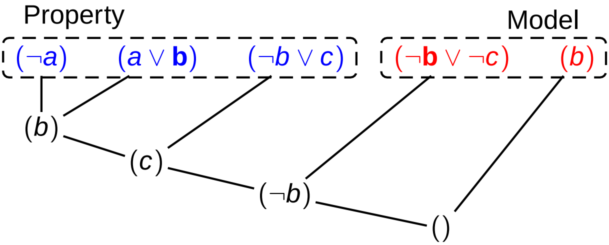

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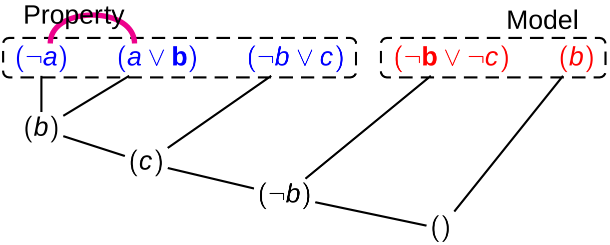

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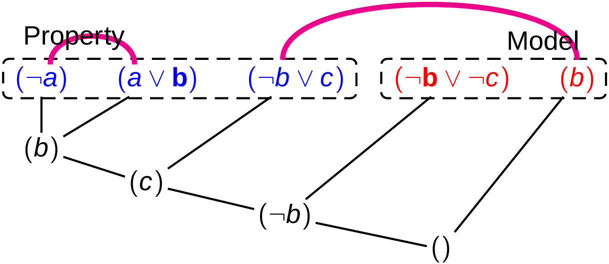

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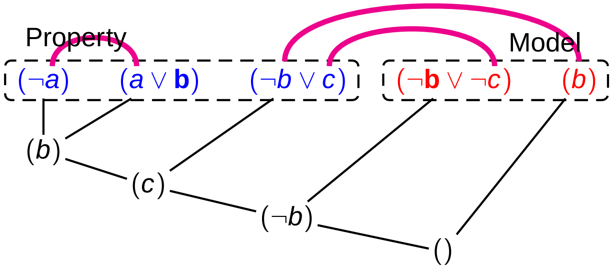

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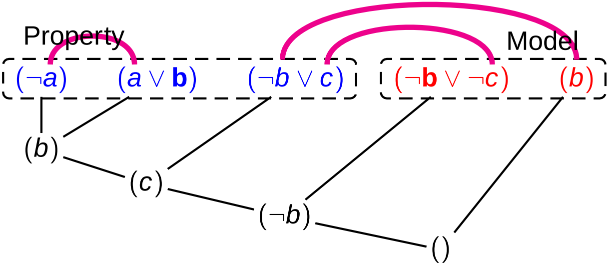

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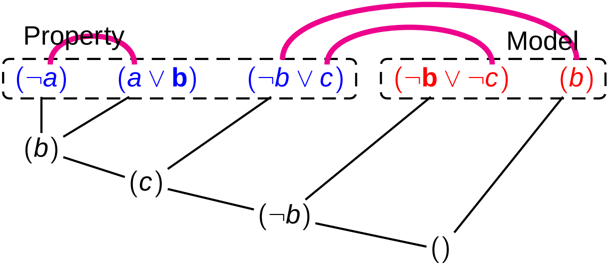

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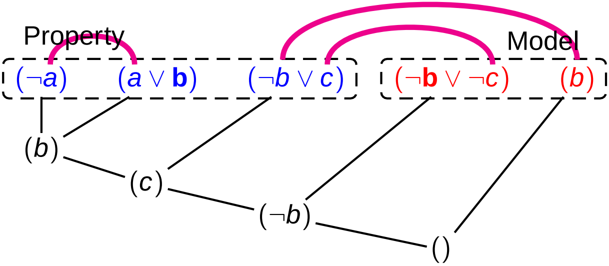

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- More precise than Irrelevance

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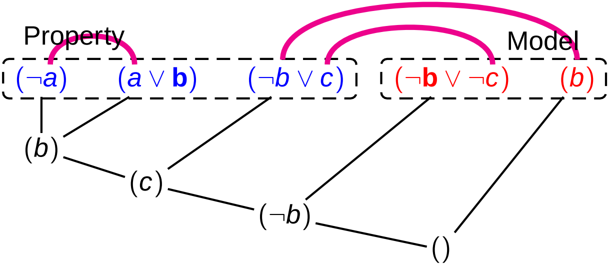

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- Still very incomplete

Algorithm 3 - Peripherality

Property

$(\neg \mathbf{a}) \quad (\mathbf{a} \vee \mathbf{b}) \quad (\neg b \vee c)$

Model

$(\neg b \vee \neg c) \quad (b)$

Variables that are not central to the proof are peripheral

Algorithm 3 - Peripherality

Property

$(\neg \mathbf{a})$ $(\mathbf{a} \vee \mathbf{b})$ $(\neg b \vee c)$

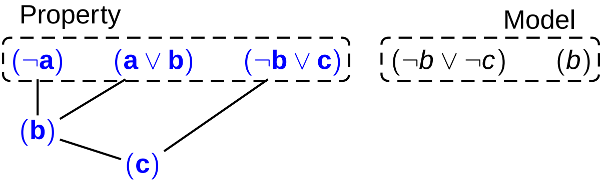
Model

$(\neg b \vee \neg c)$ (b)

$(\mathbf{b})$

Variables that are not central to the proof are peripheral

Algorithm 3 - Peripherality



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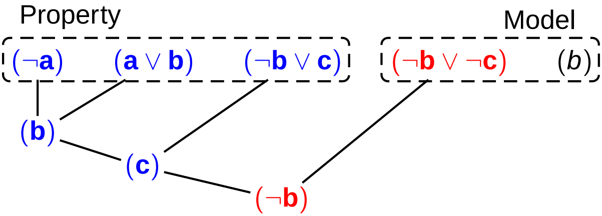

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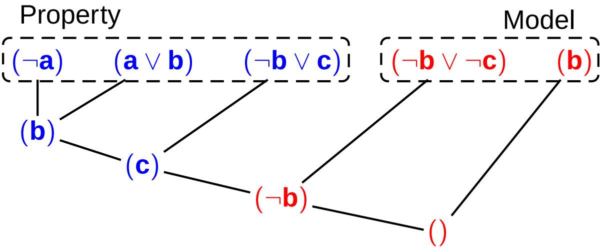

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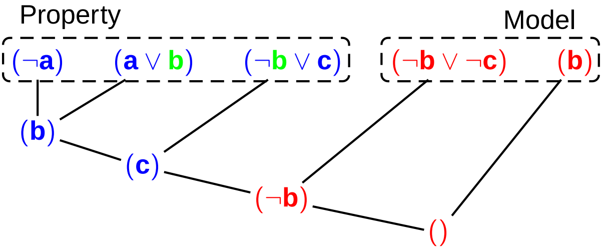

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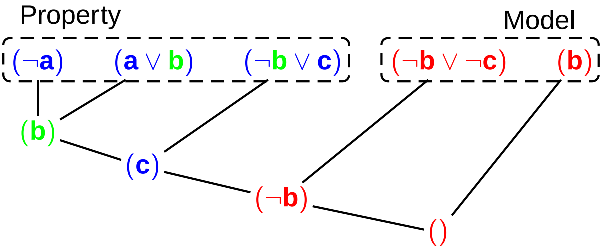

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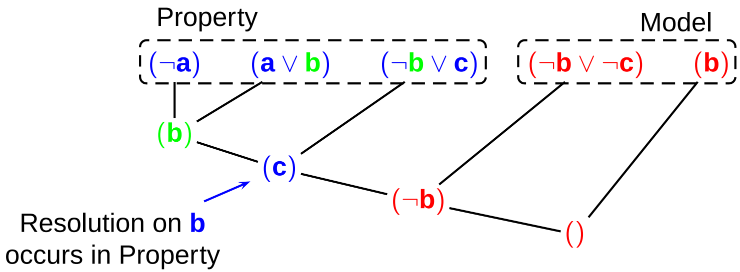

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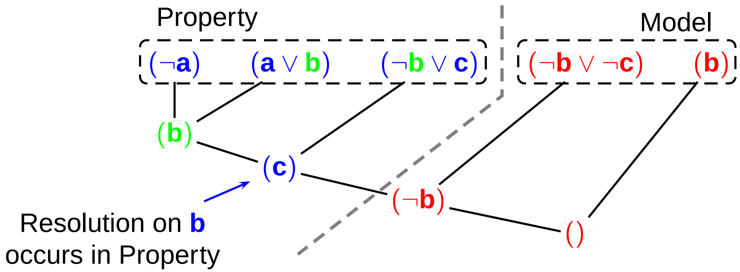

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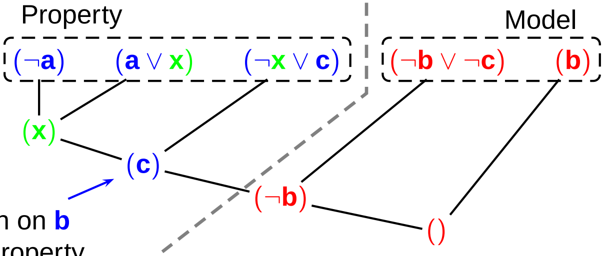

Resolution on **b** occurs in Property

Can replace **b** by **x** in Property

Variables that are not central to the proof are peripheral

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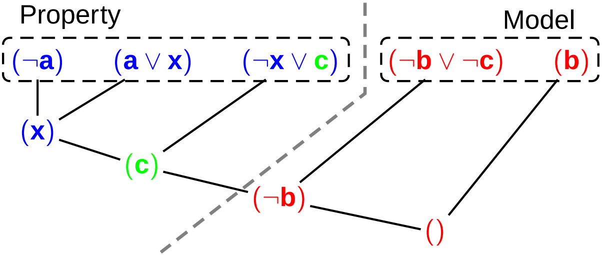

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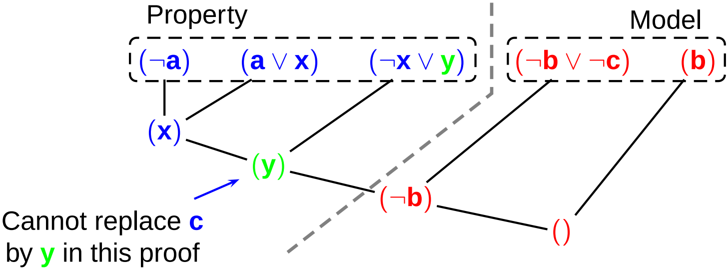

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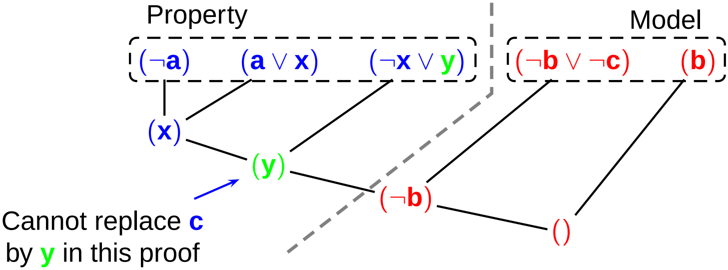

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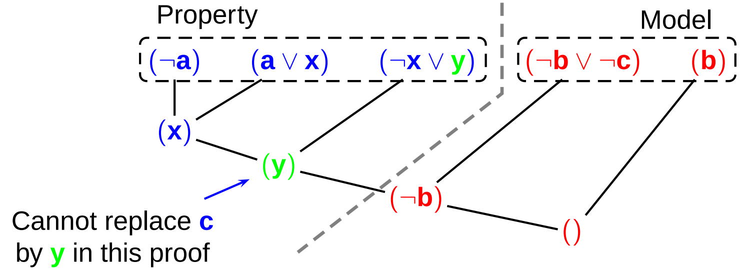

Variables that are not central to the proof are peripheral

VACUITY: replaced b by x in Property without changing proof

⇒ peripheral ⇒ vacuous

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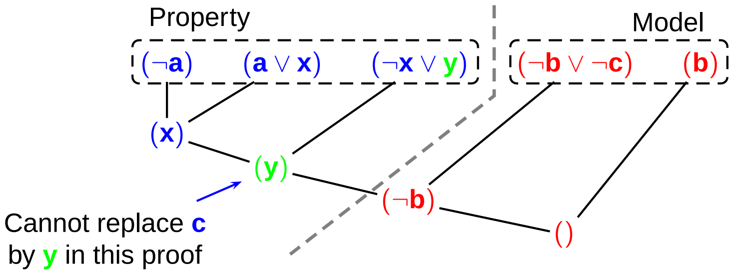

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$\Rightarrow$ peripheral $\Rightarrow$ vacuous

Linear in size of resolution proof

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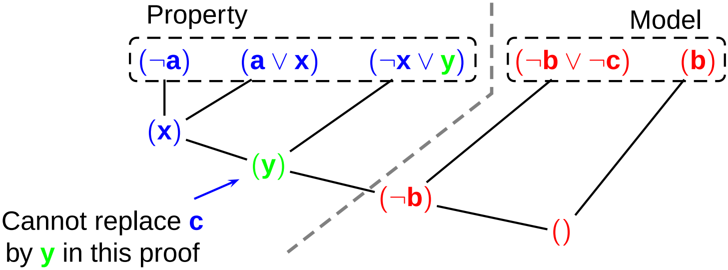

Variables that are not central to the proof are peripheral

VACUITY: replaced b by x in Property without changing proof

$\Rightarrow$ peripheral $\Rightarrow$ vacuous

- Linear in size of resolution proof
- If p is vacuous, there exists a resolution proof s.t. p is peripheral

Complete Analysis

GOAL: complete analysis using Naive Detection for leftover variables

EXAMPLE:

Model	$(\neg b \vee \neg c), (b), (\neg e), (d \vee f)$
Property	$(\neg a), (a \vee b), (\neg b \vee c), (d \vee e \vee f), (a \vee \neg c \vee d)$

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PREPROCESSING: Irrelevance algorithm

d,e,f are vacuous

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COMPLETING STEP: Naive Detection

6 extra model checking runs

$M \models p[a \leftarrow \text{true}]$?

$M \models p[a \leftarrow \text{false}]$?

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Similar for b,c

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IRRELEVANCE METHOD: Irrelevance algorithm + completing step

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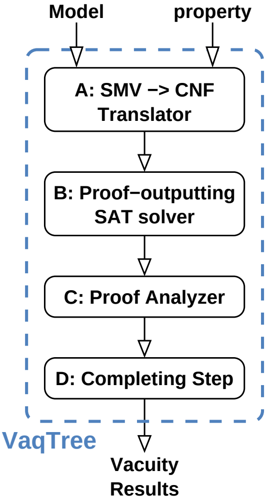
Similar for b,c

IRRELEVANCE METHOD: Irrelevance algorithm + completing step

Local Irrelevance and Peripherality are also extended in this manner

VaqTree: Vacuity Detection Framework



To our knowledge, VaqTree is the first vacuity detection tool for BMC

[A] NuSMV v. 2.3.1, modified to identify model/property clauses

[B] MiniSat-p v. 1.14, modified to output XML proof

[C] New component (Java)

- proof analysis done in memory
- 700 MB of RAM $\approx$ 2.5 million resolutions

[D] New component (Perl)

Evaluation

GOALS:

- Compare effectiveness of the three algorithms
  - how many vacuous variables can each algorithm detect?
- Evaluate the performance of the three methods, using Naive Detection as a baseline
  - are any of our methods faster than Naive Detection?

BENCHMARKS:

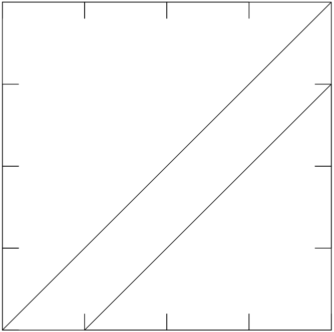
- Models and properties from the NuSMV distribution
- Models and properties from the IBM Formal Verification Benchmarks Library

Benchmark 1

Setup

- Models and properties: NuSMV distribution
- 121 properties:
  - 99 present vacuity
  - 2 - 4 temporal operators per property, from {G, F, U, X}
  - 6 variables on average, 26 max., 1 min.
- Largest proof: 2.5 million resolutions

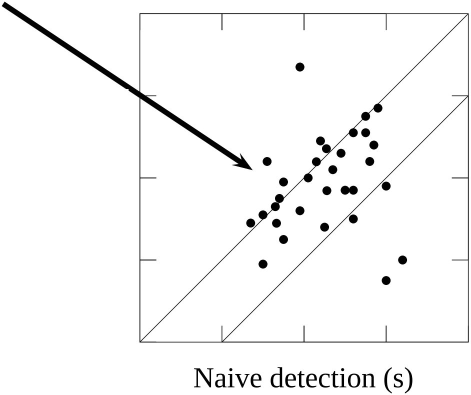
Interpreting Performance Graphs



Naive detection (s)

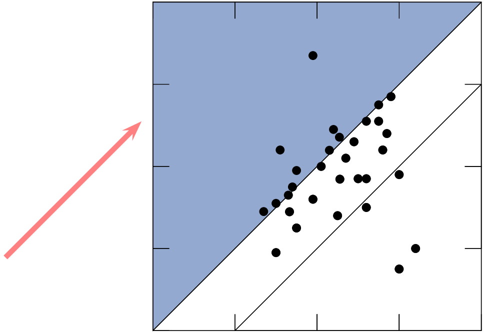
Interpreting Performance Graphs

Plotting execution times



Interpreting Performance Graphs

Naive
Detection
is faster
here



Naive detection (s)

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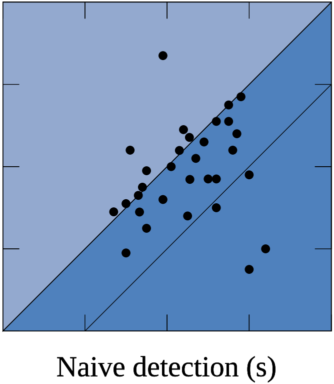
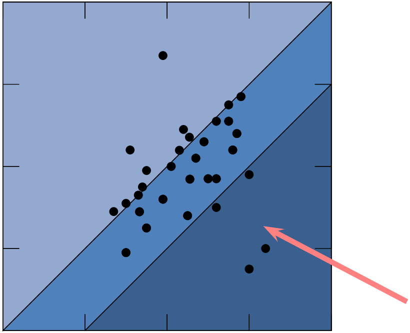


“Method” is faster here

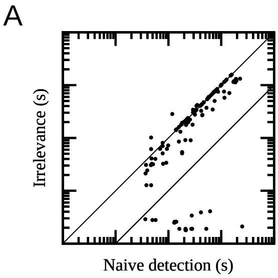
Interpreting Performance Graphs

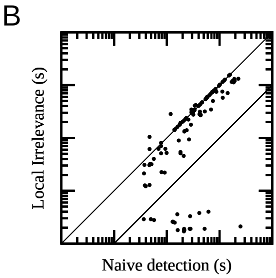


Naive detection (s)

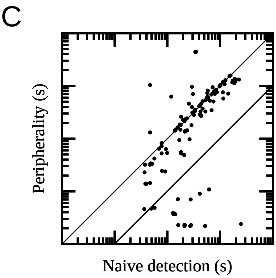
“Method” is faster by an order of magnitude here

Benchmark 1: Performance





Execution times measured for complete methods



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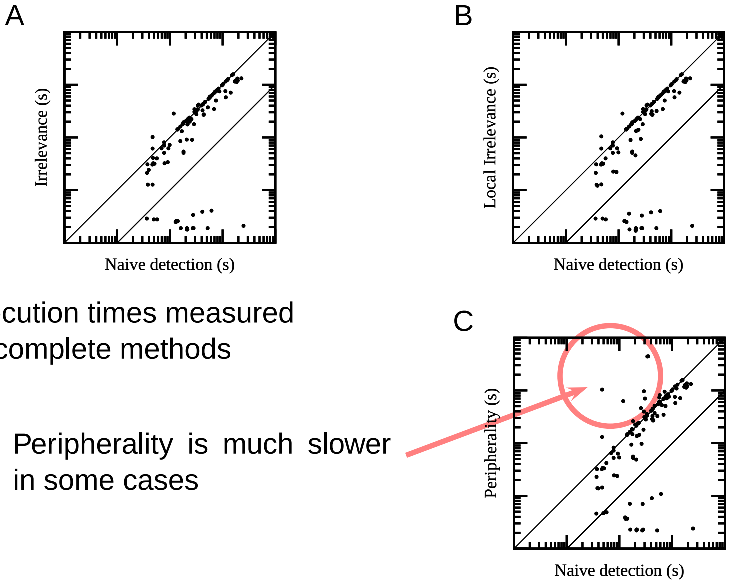
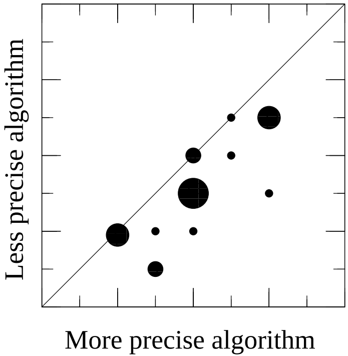


Execution times measured for complete methods

Peripherality is much slower in some cases

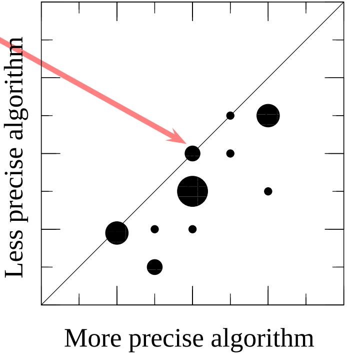
Interpreting Effectiveness Graphs



vacuous variables found: (x, y)

x = found by X-axis algorithm

y = found by Y-axis algorithm



Interpreting Effectiveness Graphs

vacuous variables found: (x, y)

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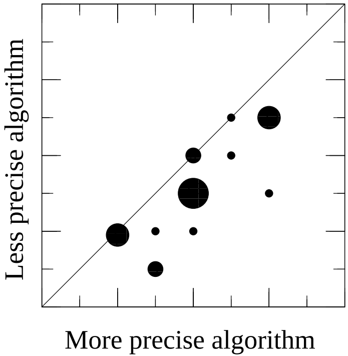
X-axis algorithm is more precise, so $x \geq y$ always



Interpreting Effectiveness Graphs

\# vacuous variables found: (x, y)

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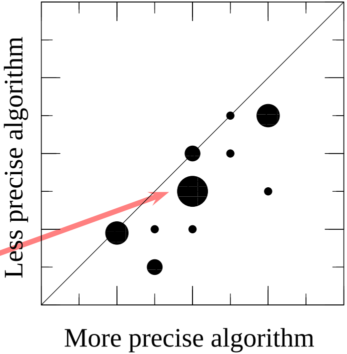
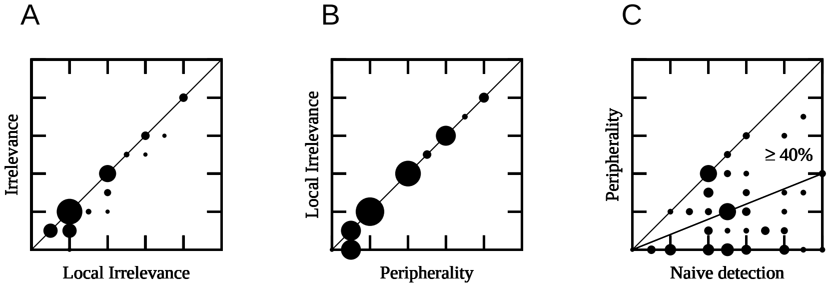
y = found by Y-axis algorithm

X-axis algorithm is more precise, so $x \geq y$ always

Larger point = more test cases



Benchmark 1: Effectiveness



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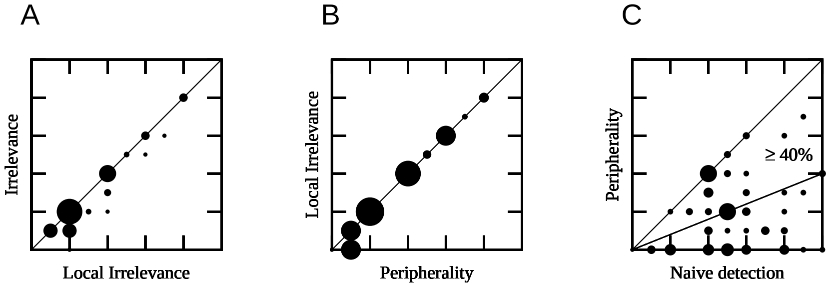

Reduced # of extra model checking runs:

- $\geq$ 40% reduction in 54% of cases with vacuity

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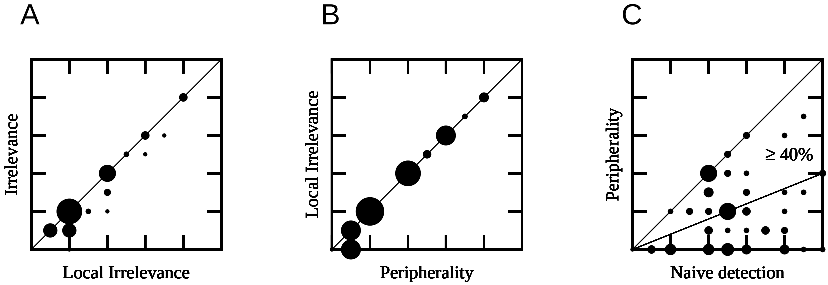

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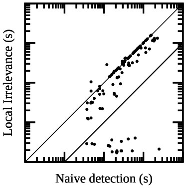
Local Irrelevance is faster than Naive Detection in 70 cases (59%):

- Twice as fast in 40% of these cases
- Order of magnitud faster in 30% of these cases



Benchmark 2

GOAL: evaluate scalability of our tool to industrial models

SETUP

- Models and properties: IBM Formal Verification Benchmarks Library
- 18 properties:
  - 12 present vacuity
  - 1 temporal operator, from $\{G, F\}$
  - 4 variables on average, 17 max., 1 min.
- Picked k-depth in line with bounds used in Benchmark 1
- Largest proof: 500k resolutions

Benchmark 2

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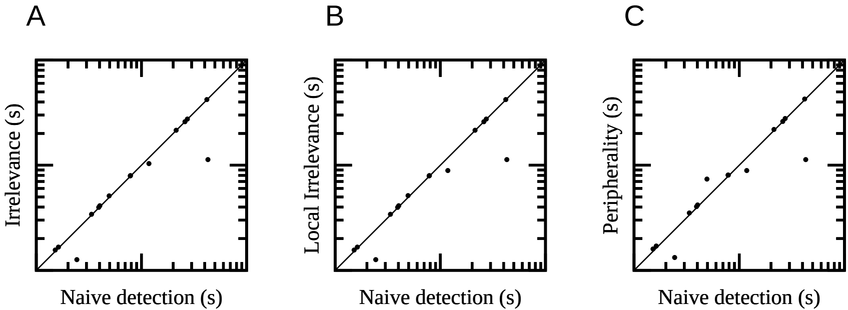
- Picked k-depth in line with bounds used in Benchmark 1
- Largest proof: 500k resolutions

Proof sizes are in same range as those for Benchmark 1

- new models are more complex
- but properties are simpler

Benchmark 2: Scalability



- Reasonable execution times
- No noticeable spike in peripherality execution times
  - models with low clause/variable ratio present vacuity
  - proofs for these models are medium-sized
- Little vacuity in this suite, yet algorithms detect some vacuity

Experimental Conclusions

	Benchmark 1	**Benchmark 2**
Models	Simple	Complex
Properties	Complex	Simple
Irrelevance	Very fast	Very fast
Local Irrelevance	Fastest	Fastest
Peripherality	Slow in certain cases	Very fast

Our algorithms:

- discover vacuous variables
- . . . via relatively inexpensive analyses of BMC artifacts

Our methods are complete and generally faster than Naive Detection

Summary

- Vacuity detection for BMC
  - we analyze BMC artifacts like UNSAT cores and resolution proofs
- Proposed and implemented a vacuity detection tool, VAQTREE

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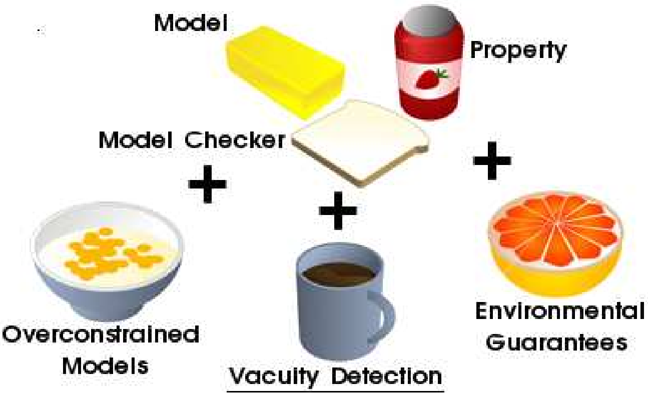
- Vacuity detection for BMC
  - we analyze BMC artifacts like UNSAT cores and resolution proofs
- Proposed and implemented a vacuity detection tool, VAQTREE
- Step towards making vacuity detection part of complete process



Future Work

- When do our algorithms apply?
  - heuristics based on clause/variable ratio and proof size
- Increase scalability of our tool
  - implement on-the-fly proof analysis
- Use interpolants for vacuity detection
- Use results of previous depths for vacuity detection

Thanks for your attention
Questions?

Why is Peripherality much slower in some cases?

Naive Detection

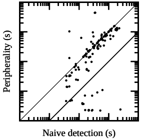
$$\Phi_1 = M \models p_1$$
$$\Phi_2 = M \models p_2$$
$$\vdots$$
$$\Phi_n = M \models p_n$$

Peripherality

$$\Phi = M \models p$$

Why is Peripherality much slower in some cases?

- Low clause/variable ratio
- No vacuous variables
- Large resolution proofs



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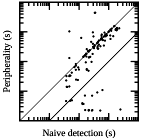

Naive Detection

$$\text{SAT} \rightarrow \begin{cases} \Phi_1 = M \models p_1 \\ \Phi_2 = M \models p_2 \\ \vdots \\ \Phi_n = M \models p_n \end{cases}$$

Peripherality

$$\Phi = M \models p \longleftarrow \text{UNSAT}$$

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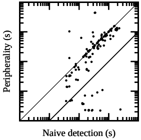

Naive Detection

SAT

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Peripherality

$$\Phi = M \models p \longleftarrow \text{UNSAT}$$

time: $\tau_1, \tau_2, \ldots \tau_n$ to find sat. assignment

T periph. analysis

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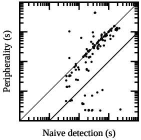

Naive Detection

SAT

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T periph. analysis

$$\tau_i <<< T$$

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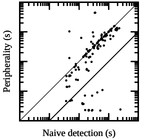

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$$\tau_i <<< T$$
$$\sum \tau_i <<< T$$