CSC411/2515 Tutorial: K-NN and Decision Tree

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Outline of This Tutorial

Cross-validation

$K$-nearest-neighbours

Decision Trees

Thanks to Ziyu Zhang and Ali Punjani for earlier versions of this tutorial.
Cross-validation

$K$-nearest-neighbours

Decision Trees
Review: Motivation for Validation

Framework: learning as optimization

Goal: optimize model complexity (for our task)

Formulation: minimize underfitting and overfitting
Review: Motivation for Validation

Framework: learning as optimization

Goal: optimize model complexity (for our task)

Formulation: minimize underfitting and overfitting

In particular, we want our model to generalize well without overfitting.
Review: Motivation for Validation

Framework: learning as optimization

Goal: optimize model complexity (for our task)

Formulation: minimize underfitting and overfitting

In particular, we want our model to generalize well without overfitting.

We can ensure this by validating the model.
Types of Validation (1)

hold-out validation: split data into training set and validation set

▶ usually: 30% as hold-out set

![Diagram showing original set split into training and testing sets.](Image)
Types of Validation (1)

**hold-out validation**: split data into training set and validation set

- usually: 30% as hold-out set

**Problems:**

- waste of dataset
- estimation of error rate may be misleading
Types of Validation (2)

cross-validation: random sub-sampling

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Figure from Bishop, C. M. (2006). *Pattern Recognition and Machine Learning*. Springer.

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Types of Validation (2)

cross-validation: random sub-sampling

Problem:

- more computationally expensive than *hold-out* validation

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1 Figure from Bishop, C. M. (2006). *Pattern Recognition and Machine Learning*. Springer.
Variants of Cross-validation (1)

leave-$p$-out: use $p$ examples as the validation set, and the rest as training; repeat for all configurations of examples.

e.g., for $p = 1$:
Variants of Cross-validation (1)

leave-$p$-out: use $p$ examples as the validation set, and the rest as training; repeat for all configurations of examples.

e.g., for $p = 1$:

Problem:

- exhaustive: We are required to train and test \( \binom{N}{p} \) times, where $N$ is the number of training examples.
Variants of Cross-validation (2)

\textbf{K-fold:} partition training data into \( K \) equally sized subsamples; for each fold, use \( K - 1 \) subsamples as training data with the last subsample as validation.
Variants of Cross-validation (2): $K$-fold

Advantages:

- All observations are used for both training and validation, and each observation is used for validation exactly once.
- non-exhaustive $\implies$ more tractable than LpOCV

Problems:

- expensive for large $N$, $K$ (since we train/test $K$ models on $N$ examples)
- but there are some efficient hacks to save time over the brute-force method...
- can still overfit if we validate too many models!

Solution: hold out an additional test set before doing any model selection, and check that the best model performs well even on the additional test set (nested cross-validation)
Variants of Cross-validation (2): \( K \)-fold

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- \textbf{expensive} for large \( N, K \) (since we train/test \( K \) models on \( N \) examples)
  - but there are some efficient hacks to save time over the brute-force method . . .
- can still \textbf{overfit} if we validate too many models!
  - \textbf{Solution}: hold out an additional test set before doing any model selection, and check that the best model performs well even on the additional test set (\textit{nested cross-validation})
Practical Tips for Using $K$-fold Cross-validation

▶ **Q:** How many folds do we need?

▶ **A:** with larger $K$, . . .

  ▶ error estimation tends to be more accurate

  ▶ but computation time will be greater
Practical Tips for Using $K$-fold Cross-validation

▶ Q: How many folds do we need?
▶ A: with larger $K$, ...
  ▶ error estimation tends to be more accurate
  ▶ but computation time will be greater

In practice:
▶ usually choose $K \approx 10$
▶ BUT larger dataset $\implies$ choose smaller $K$
Cross-validation

$K$-nearest-neighbours

Decision Trees
**K-nearest-neighbours: Definition**

**Training:** store all training examples (perfect memory)

**Test:** predict value/class of an unseen (test) instance based on closeness to stored training examples, relative to some distance (similarity) measure

\[\text{Figure from Murphy, K. P. (2012). Machine Learning: A Probabilistic Perspective. MIT press.}\]
Predicting with \( K \)-nearest-neighbours

- for \( K = 1 \),
  - predict the same value/class as the nearest instance in the training set.

- for \( K > 1 \),
  - find the \( K \) closest training examples, and either
    - predict class by majority vote (in classification).
    - predict value by average weighted inverse distance (in regression).
Practical Tips for Using *KNN* (1)

- ties may occur in a classification problem when $K > 1$
  - for binary classification: choose $K$ odd to avoid ties
  - for multi-class classification:
    - decrease the value of $K$ until the tie is broken
    - if that doesn’t work, use the class given by a 1NN classifier
Practical Tips for Using KNN (2)

- magnitude of $K$:
  - smaller $K$: predictions have higher variance (less stable)
  - larger $K$: predictions have higher bias (less true)
Aside: The Bias-variance Tradeoff

A learning procedure creates **biased** models if . . .

- the predictive distribution of the models differs greatly from the target distribution.
Aside: The Bias-variance Tradeoff

A learning procedure creates biased models if...

- the predictive distribution of the models differs greatly from the target distribution.

A learning procedure creates models with high variance if...

- the models have greatly different test predictions (across different training sets from the same target distribution).
Practical Tips for Using KNN (2)

- magnitude of $K$:
  - smaller $K$: predictions have higher variance (less stable)
  - larger $K$: predictions have higher bias (less true)

Cross-validation can help here!
Practical Tips for Using KNN (3)

- the choice of distance measure affects the results!
  - e.g., standard Euclidean distance:

\[
d_E(x, y) = \sqrt{\sum_i (x_i - y_i)^2}
\]

Problems:

- assumes features have equal variance
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  - **Solution**: standardize / scale the feature values

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- in high-dimensional space, noisy features dominate
Practical Tips for Using KNN (3)

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- assumes features are uncorrelated
  - **Solution**: use a more complex model (e.g., a Gaussian)

- in high-dimensional space, noisy features dominate
  - **Solution**: apply (learn?) feature weightings
Practical Tips for Using KNN (4)

- KNN has perfect memory, so computational complexity is an issue
  - at test time: $O(N \cdot D)$ computations per test point
Practical Tips for Using KNN (4)

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  - at test time: $O(N \cdot D)$ computations per test point

Solutions:

- dimensionality reduction
  - sample features
    - project the data to a lower dimensional space
  - sample training examples
  - use clever data structures, like $k$-D trees
MATLAB Demo
Cross-validation

\(K\)-nearest-neighbours

Decision Trees
Decision Trees: Definition

Goal: Approximate a discrete-valued target function

Representation: a tree, of which

- each internal (non-leaf) node tests an attribute
- each branch corresponds to an attribute value
- each leaf node assigns a class

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The ID3 algorithm:

- while training examples are not perfectly classified, do
  - choose the “most informative” attribute $\theta$ (that has not already been used) as the decision attribute for the next node $N$ (greedy selection)
  - for each value (discrete $\theta$) / range (continuous $\theta$), create a new descendant of $N$
  - sort the training examples to the descendants of $N$
## Decision Trees: Example *PlayTennis*

<table>
<thead>
<tr>
<th>Day</th>
<th>Outlook</th>
<th>Temperature</th>
<th>Humidity</th>
<th>Wind</th>
<th>PlayTennis</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>Sunny</td>
<td>Hot</td>
<td>High</td>
<td>Weak</td>
<td>No</td>
</tr>
<tr>
<td>D2</td>
<td>Sunny</td>
<td>Hot</td>
<td>High</td>
<td>Strong</td>
<td>No</td>
</tr>
<tr>
<td>D3</td>
<td>Overcast</td>
<td>Hot</td>
<td>High</td>
<td>Weak</td>
<td>Yes</td>
</tr>
<tr>
<td>D4</td>
<td>Rain</td>
<td>Mild</td>
<td>High</td>
<td>Weak</td>
<td>Yes</td>
</tr>
<tr>
<td>D5</td>
<td>Rain</td>
<td>Cool</td>
<td>Normal</td>
<td>Weak</td>
<td>Yes</td>
</tr>
<tr>
<td>D6</td>
<td>Rain</td>
<td>Cool</td>
<td>Normal</td>
<td>Strong</td>
<td>No</td>
</tr>
<tr>
<td>D7</td>
<td>Overcast</td>
<td>Cool</td>
<td>Normal</td>
<td>Strong</td>
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<tr>
<td>D8</td>
<td>Sunny</td>
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<td>High</td>
<td>Weak</td>
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<tr>
<td>D9</td>
<td>Sunny</td>
<td>Cool</td>
<td>Normal</td>
<td>Weak</td>
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<tr>
<td>D10</td>
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<tr>
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<tr>
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<td>High</td>
<td>Strong</td>
<td>Yes</td>
</tr>
<tr>
<td>D13</td>
<td>Overcast</td>
<td>Hot</td>
<td>Normal</td>
<td>Weak</td>
<td>Yes</td>
</tr>
<tr>
<td>D14</td>
<td>Rain</td>
<td>Mild</td>
<td>High</td>
<td>Strong</td>
<td>No</td>
</tr>
</tbody>
</table>
After splitting the training examples first on *Outlook* . . .

What should we choose as the next attribute under the branch *Outlook* = Sunny?
Choosing the “Most Informative” Attribute

Formulation: Maximise information gain over attributes $Y$.

Information Gain ($PlayTennis \mid Y$)

\[ = H(PlayTennis) - H(PlayTennis \mid Y) \]

\[ = \sum_x P(PlayTennis = x) \log P(PlayTennis = x) \]

\[ - \sum_{x,y} P(PlayTennis = x, Y = y) \log \frac{P(Y = y)}{P(PlayTennis = x, Y = y)} \]
### Information Gain Computation (1)

\[
\text{InfoGain (} \textit{PlayTennis} \mid \textit{Humidity}) = 0.970 - \frac{3}{5}(0.0) - \frac{2}{5}(0.0) = 0.970
\]

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Information Gain Computation (2)

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\text{InfoGain} (PlayTennis \mid \text{Temperature}) = 0.970 - \frac{2}{5}(0.0) - \frac{2}{5}(1.0) - \frac{1}{5}(0.0) = 0.570
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Information Gain Computation (3)

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\[
\text{InfoGain (\textit{PlayTennis} \mid \textit{Wind})} = 0.970 - \frac{2}{5}(1.0) - \frac{3}{5}(0.918) = 0.019
\]
The Decision Tree for *PlayTennis*
Recall: The Bias-variance Tradeoff

Where do decision trees naturally lie in this space?

Answer: high variance

How to fix: pruning (e.g., reduced-error pruning, rule post-pruning)
Recall: The Bias-variance Tradeoff

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Recall: The Bias-variance Tradeoff

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