Verification of Uncertainty Reducing Model Transformations

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Abstract. Models are typically used for expressing information that is known at a particular stage in the software development process. Yet, it is also important to express what information a modeler is still uncertain about. Furthermore, when a transformation is applied to a model containing uncertainty, it is natural to consider the effect that the transformation has on the level of uncertainty, e.g., whether it always reduces it. In previous work, we have presented a general approach for precisely expressing uncertainty within models and defined formal conditions for uncertainty reduction between models. In this paper, we use these foundations to develop an automated method for verifying that a partial model transformation is uncertainty reducing.

1 Introduction

Software modelers often face the challenging task of working in the presence uncertainty. Uncertainty in models comes from many sources, including but not limited to incomplete requirements [8], presence of alternative design decisions [31], disagreements among stakeholders [24], etc. Yet existing modeling methodologies, languages and tools rarely provide adequate support for explicating uncertainty. To help address this gap in expressiveness, we have proposed several types of *partiality* annotations with formal semantics that could be used to augment any modeling language with the means to accurately express uncertainty [27]. We call the resulting models *partial*.

In [10], we argued that ubiquity of uncertainty should make partial models first-class artifacts in MDE, allowing these models to be manipulated "as usual": transformed, checked for correctness, etc. In [12], we started looking at how to lift classical transformations to apply to partial models, and in [11], we conducted experiments using partial models for property checking.

In this paper, we are concerned with transformations that take the form of graph rewriting *rules* [9]. Such a rule is applied by finding a match of its left hand side (LHS) in the source model and replacing it with the fragment on its right hand side (RHS). A transformation corresponds to repeated applications of the rule until it can no longer be applied.

Once points of uncertainty are explicated, uncertainty resolution is reflected in the construction of a *partial model refinement* of the model. Yet, natural language expression of uncertainty makes it impossible to accurately (or automatically) verify the correctness of refinement.

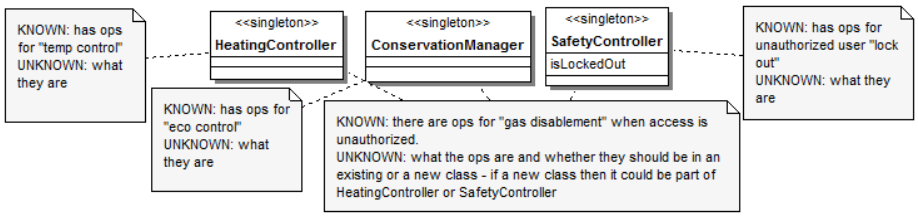


Fig. 1. Class diagram for HVAC example showing ad-hoc expressions of uncertainty.

Our formalization [27] allows us to check whether a particular rule-based transformation is a partial model refinement, i.e., that it reduces the level of uncertainty. We distinguish between two types of questions: given a rule, (1) determine whether a particular instance of applying it to a given model is correct, i.e., that it reduces uncertainty – we call this verifying correctness of refinement *application*, and (2) determine whether the rule, when applied to *all* models, is correct – we call this verifying correctness of a refining *transformation*.

Motivating example. The partial modeling approach can be applied to any modeling language. In this example we use a simplified version of UML class diagrams. Assume a scenario, depicted in Figure 1, where a modeler is facing uncertainty regarding a fragment of the class diagram in a hypothetical HVAC (Heating, Ventilation, A/C) controller for a building. The `HeatingController` has operations to regulate the building's temperature and `ConservationManager` has operations to monitor consumption and conserving energy. A separate class `SafetyController` interfaces with the Security subsystem of the building, and so has operations to detect HVAC-specific malicious intrusions. The modeler also knows that there should exist operations for disabling the gas supply for the building (e.g, in case of a fire or a leak, etc) but is not sure where they belong or whether they should be in a separate class, etc.

The textual notes in the diagram represent the modeler's uncertainty by stating specific information that is known and unknown about the model. All of this information must be specified ad-hoc, using natural language, since there is no notational mechanism in the class diagram language for it.

Model `P1` in Figure 1 shows the use of partiality annotations, introduced in [27], to express the uncertainty in Figure 1. In each case, the annotation is given in brackets as a prefix to the element's name. For example, the S annotation on the operation `ecoControlOps` (in class `ConservationManager`) means that it represents a (as yet unknown) set of operations. This captures the same information as in the note attached to `ConservationManager` in Figure 1 – i.e., that it contains operations for energy conservation but it is still unknown what they are. The V annotation on the `GasDisabler` class means that it is a "variable" class and that it is still unknown whether it is assigned to a new class or to one of the existing classes; however, regardless of how it gets assigned, it must contain a set of `gasDisablerOps` operations. Furthermore, the M-annotated composition associations say that if `GasDisabler` is assigned to a new class then it *may* have

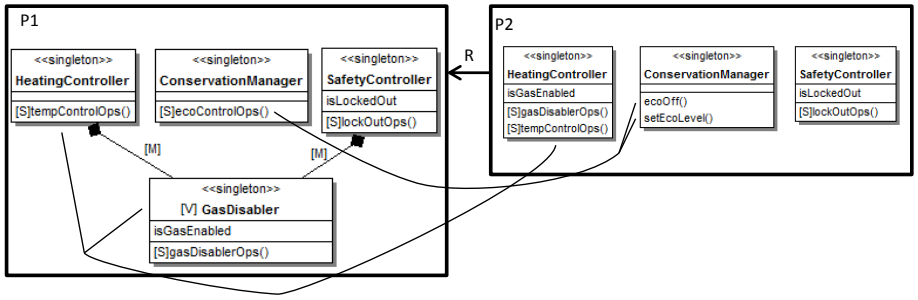


Fig. 2. Example refinement of the partial model `P1`.

a composition relationship either with the `HeatingController` class or with the `SafetyController` class. Yet it cannot have this relationship with both classes simultaneously since the well-formedness rules for class diagrams prohibit this.

As more information becomes available, the modeler can resolve some of these uncertainties by constructing a *partial model refinement* of the original model. For example, Figure 1 shows a partial model refinement of the partial class diagram `P1`. The refinement represents the way in which the elements in the two models are mapped to each other and captures the uncertainty resolution decisions made. To avoid visual clutter, we show only the non-obvious parts of the mapping: (1) the S-annotated operation `ecoControlOps`() is refined to a set of particular operations {`ecoOff`(), `setEcoLevel`()}, (2) a decision is made to put the functionality to disable the gas supply into the `HeatingController` class by assigning the V-annotated class `GasDisabler` to it, and (3) the M-composition relations are eliminated.

Figure 1 shows a refinement *application*, i.e., a refinement *applied to a particular model*. In contrast, Figure 3 gives a refining partial model transformation that can be used to generate a refinement application when *applied to an arbitrary model*. Syntactically, *ReduceAbs* removes all occurrences of S annotations on elements. Semantically, it means that these now represent particular elements rather than an arbitrary set of elements. Intuitively, this transformation reduces uncertainty about these elements and thus is an uncertainty-reducing transformation. But can we prove it?

Contributions of this paper. In this paper, we look at the problem of checking correctness of uncertainty-reducing transformations of partial models, such as *ReduceAbs*. In previous work [25], we have presented a general approach for precisely expressing uncertainty within models and defined formal conditions for uncertainty reduction between models. We then showed how to prove that a *particular application* of a rule is uncertainty-reducing, i.e., that applying a given rule to a given model results in a refinement of this model.

In this paper, we use these foundations to develop an automated method for showing that a partial model *transformation* is uncertainty reducing, i.e., given a set of rules, applying it to *any model* yields an uncertainty-reducing refinement. Specifically, we make the following contributions: (1) we develop and illustrate an automated method for verifying partial model refinement transformations;

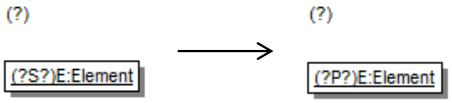


Fig. 3. A rule defining transformation *ReduceAbs*.

(2) we describe prototype tool support to help automate the verification method and to use the generated counterexamples in order to repair faulty rules; and (3) we apply the method to the verification of three specific transformations. Our tool support, all models and proof of Proposition 1 are available at `http://www.cs.toronto.edu/~jgorzny`.

The rest of the paper is organized as follows. In Section 2, we review the concept of model partiality as introduced in [27,25]. In Section 3, we define an automated method for verifying uncertainty-reducing transformations, which consists of checking two properties. Sections 4 and 5 describe the theoretical foundations and the tool support for automating checking each of these properties. In Section 7, we apply the method on several example transformations. In Section 8, we discuss related work. In Section 9, we summarize the paper and discuss potential future research directions.

2 Background

In this section, we briefly review the concepts of language-independent partial modeling introduced in [27].

***MAVO* partial models.** When a model contains partiality information, we call it a *partial* model. Semantically, a partial model represents the set of different possible *concrete* (i.e., non-partial) models that would resolve the uncertainty represented by the partiality. More formally:

Definition 1 (Partial model) *A* partial model P *consists of a* base model*, denoted* $bs(P)$*, and a set of annotations. Let* T *be the metamodel of* $bs(P)$*. Then,* $[P]$ *denotes the set of* T *models called the* concretizations *of* P*.* P *is called consistent iff* $[P] \neq \emptyset$*.*

For example, Figure 4(a) shows the base model (i.e., what remains when the annotations are stripped away) of the partial class diagram `P1` in Figure 1. Note that the base model does not necessarily need to be well-formed. In fact, the example in the figure violates the well-formedness rule that a singleton class cannot be composed into two different classes. This shows that expressing some cases of uncertainty requires non-well-formed base models. Figure 4(b) shows one of the concretizations of `P1`. `P1` has an infinite number of concretizations since each of the S-annotated operations can be replaced by any set of particular operations. Thus, although bs(`P1`) is not well-formed, `P1` is still consistent since it has concretizations.

We use four types of partiality annotations, each adding support for a different type of uncertainty in a model: Annotating an element with M indicates

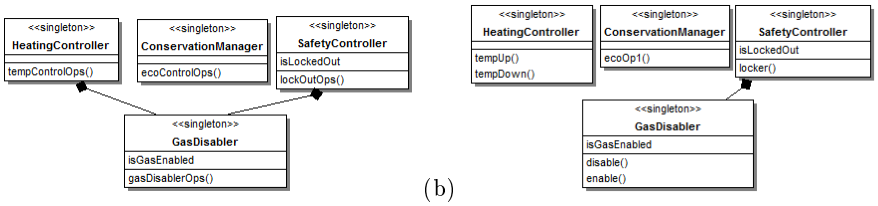
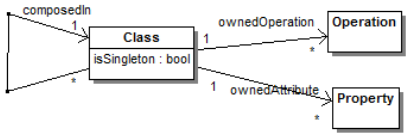


Fig. 4. The base model (a) and a concretization (b) of the partial class diagram `P1` in Figure 1.



Additional constraints:

(1) A singleton class cannot be composed in more than one class.

$\forall c\colon Class \cdot \exists c_1, c_2\colon Class \cdot isSingleton(c) \wedge composedIn(c, c_1) \wedge composedIn(c, c_2) \Rightarrow c_1 = c_2$

Fig. 5. A simplified metamodel of the UML class diagram language.

that we are unsure about whether it should exist in the model while E (default) means that it does exist; annotating an element with S indicates that we are unsure about whether it should actually be a collection of elements while P (default) means that it is just one element; annotating an element with V indicates that we are unsure about whether it should actually be merged with other elements while C (default) means that it is distinct; finally, annotating the entire model with INC indicates that we are unsure about whether it is complete while COMP (default) means that it is complete. When these four types of partiality annotations are used together, we refer to it as *MAVO* partiality.

Formalizing *MAVO* partiality. A metamodel represents a set of models and can be expressed as a First Order Logic (FOL) theory.

Definition 2 (Metamodel) *A* metamodel *is an FOL theory $T = \langle \Sigma, \Phi \rangle$, where Σ is the* signature *with sorts and predicates representing the element types, and Φ is a set of sentences representing the well-formedness constraints. The models that conform to T are the finite FO Σ-structures that satisfy Φ according to the usual FO satisfaction relation. We denote the set of models with metamodel T by $Mod(T)$.*

The simple class diagram metamodel in Figure 5 fits this definition if we interpret boxes as sorts and edges as predicates comprising $\Sigma_{\texttt{CD}}$ (where `CD` stands for "class diagram") and take the multiplicity constraints (translated to FOL) and the additional constraint (1) as comprising $\Phi_{\texttt{CD}}$.

Like a metamodel, a partial model also represents a set of models and thus can also be expressed as an FOL theory. Specifically, for a partial model P, we construct a theory $FO(P)$ s.t. $Mod(FO(P)) = [P]$. We proceed as follows. (1) Let $M = bs(P)$ be the base model of a partial model P. We define a new

$\Sigma_{\texttt{M1}}$ has unary predicates $\texttt{CM}(\texttt{Class})$, $\texttt{ECOps}(\texttt{Operation})$), ...,
and binary predicates $\texttt{CMownsECOps}(\texttt{Class}, \texttt{Operation})$, ...
$\Phi_{\texttt{M1}}$ contains the following sentences:
(Complete) $(\forall x : \texttt{Class} \cdot \texttt{CM}(x) \vee \texttt{HC}(x) \vee \texttt{SC}(x) \vee \texttt{GD}(x)) \wedge$
$(\forall x : \texttt{Class}, y : \texttt{Operation} \cdot \texttt{ownedOperation}(x, y)$
$\Rightarrow (\texttt{CMownsECOps}(x, y) \vee \ldots)) \wedge \ldots$
CM:
$(\mathit{Exists}_{\texttt{CM}})$ $\exists x : \texttt{Class} \cdot \texttt{CM}(x)$
$(\mathit{Unique}_{\texttt{CM}})$ $\forall x, x' : \texttt{Class} \cdot \texttt{CM}(x) \wedge \texttt{CM}(x') \Rightarrow x = x'$
$(\mathit{Distinct}_{\texttt{CM}-\texttt{HC}})$ $\forall x : \texttt{Class} \cdot \texttt{CM}(x) \Rightarrow \neg\texttt{HC}(x)$
$(\mathit{Distinct}_{\texttt{CM}-\texttt{SC}})$ $\forall x : \texttt{Class} \cdot \texttt{CM}(x) \Rightarrow \neg\texttt{SC}(x)$
$(\mathit{Distinct}_{\texttt{CM}-\texttt{GD}})$ $\forall x : \texttt{Class} \cdot \texttt{CM}(x) \Rightarrow \neg\texttt{GD}(x)$
similarly for all other element and relation predicates

Fig. 6. The FO encoding of $P_{\texttt{M1}}$.

partial model P_M which has M as its base model and its sole concretization, i.e., $bs(P_M) = M$ and $[P_M] = \{M\}$. We call P_M the *ground* model of P. (2) To construct the FOL encoding of P_M, $FO(P_M)$, we extend T to include a unary predicate for each element in M and a binary predicate for each relation instance between elements in M. Then, we add constraints to ensure that the only first order structure that satisfies the resulting theory is M itself. (3) We construct $FO(P)$ from $FO(P_M)$ by removing constraints corresponding to the annotations in P. This constraint relaxation allows more concretizations and so represents increasing uncertainty. For example, if an atom a in P is annotated with M then the constraint that enforces occurrence of a in every concretization is removed. We illustrate the above construction using the partial class diagram P1 in Figure 1.

(1) Let $\texttt{M1} = bs(\texttt{P1})$ be its base model and $P_{\texttt{M1}}$ be the corresponding ground partial model.

(2) We have: $FO(P_{\texttt{M1}}) = \langle \Sigma_{\texttt{CD}} \cup \Sigma_{\texttt{M1}}, \Phi_{\texttt{CD}} \cup \Phi_{\texttt{M1}} \rangle$ (see Definition 2), where $\Sigma_{\texttt{M1}}$ and $\Phi_{\texttt{M1}}$ are model M1-specific predicates and constraints, defined in Figure 6. They extend the signature and constraints for CD models described in Figure 5. For conciseness, we abbreviate element names in Figure 6, e.g., ConservationManager becomes CM, etc. We refer to $\Sigma_{\texttt{M1}}$ and $\Phi_{\texttt{M1}}$ as the *MAVO* predicates and constraints, respectively.

Since $FO(P_{\texttt{M1}})$ extends CD, the FO structures that satisfy $FO(P_{\texttt{M1}})$ are the class diagrams that satisfy the constraint set $\Phi_{\texttt{M1}}$ in Figure 6. Assume N is such a class diagram. The *MAVO* constraint *Complete* ensures that N contains no more elements or relation instances than M1. Now consider the class CM in M1. $\mathit{Exists}_{\texttt{CM}}$ says that N contains at least one class called CM, $\mathit{Unique}_{\texttt{CM}}$ – that it contains no more than one class called CM, and the clauses $\mathit{Distinct}_{\texttt{CM}-*}$ – that the class called CM is different from all the other classes. Similar *MAVO* constraints are given for all other elements and relation instances in M1. These constraints ensure that $FO(P_{\texttt{M1}})$ has exactly one concretization and thus $N = \texttt{M1}$.

(3) Relaxing the *MAVO* constraints $\Phi_{\texttt{M1}}$ allows additional concretizations and represents a type of uncertainty indicated by a partiality annotation. For example, if we use the INC annotation to indicate that `M1` is incomplete, we can express this by removing the *Complete* clause from $\Phi_{\texttt{M1}}$ and thereby allow concretizations to be class diagrams that extend `M1`. Similarly, expressing the effect of the M, S and V annotations for an element E correspond to relaxing $\Phi_{\texttt{M1}}$ by removing Exists_E, Unique_E and $\mathit{Distinct}_{E-*}$ clauses, respectively. For example, removing the $\mathit{Distinct}_{\texttt{GD}-*}$ clauses is equivalent to marking the class `GD` with V(i.e., `GasDisabler` may or may not be distinct from another class).

Partial model refinement. Intuitively, refinement of a model should not increase the set of concretizations it has (a *proper* refinement reduces this), while making sure at least one concretization remains. In what follows, assume that there exists a refinement mapping R which maps the atoms of the two models.

Definition 3 (*MAVO* Mapping) *Given MAVO models P and P', based on the same metamodel, a* MAVO mapping $R(P, P')$ *is a relation $R \subseteq atoms(P) \times atoms(P')$, where $atoms(P)$ and $atoms(P')$ are the sets of atoms in P and P', respectively, and for all $\langle a, a' \rangle \in R$, a' and a have the same type in the metamodel.*

We now define a notion of a *simple extension* of a *MAVO* mapping:

Definition 4 (Simple extension) *A* MAVO *mapping $R_1(P_1, P_1')$ is a* simple extension *of a mapping $R(P, P')$ iff $R_1(P_1, P_1')$ is constructed by adding the same set of annotated atoms to both P and P' to form P_1 and P_1', respectively, and adding the corresponding identity mappings to R to form R_1.*

We now review, following [25], how the FO encoding is used to formalize the above intuition by giving two conditions for correctness of *MAVO* refinement, i.e., conditions under which it reduces uncertainty.

Definition 5 (*MAVO* Refinement) *Let $R(P, P')$ be a* MAVO *mapping where we have encodings $FO(P') = \langle \Sigma_{P'}, \Phi_{P'} \rangle$ and $FO(P) = \langle \Sigma_P, \Phi_P \rangle$. P'* refines *P with mapping R iff the following conditions hold:*

(Ref1) $\Phi_{P'}$ is satisfiable
(Ref2) $\Phi_{P'} \Rightarrow R(\Phi_P)$

R is then called a refinement mapping.

Condition Ref1 ensures that P' has at least one concretization (see Definition 1) and condition Ref2 ensures that P has all concretizations of P'. Here, $R(\Phi_P)$ denotes a translation (described in [25]) of the *MAVO* sentences of P according to the mapping R.

3 Verifying Uncertainty Reducing *MAVO* Transformations

Definition 5 in Section 2 defined conditions for verifying a *single application* of partial model refinement. In this section, we present a method for verifying that

every input/output pair of a partial model transformation is a valid refinement application (i.e., satisfies Definition 5). We call such a transformation *refining*.

We assume that the partial models are specified using *MAVO* annotations described in Section 2 and express the conditions for a refining *MAVO* transformation as follows.

Definition 6 (Refining *MAVO* Transformation) *A MAVO transformation* $F : Mod(MAVO(T)) \rightarrow Mod(MAVO(T))$ *is a* refining *transformation iff the following conditions hold for all* $P \in Mod(MAVO(T))$ *where* $FO(P) = \langle \Sigma_P, \Phi_P \rangle$, $FO(F(P)) = \langle \Sigma_{F(P)}, \Phi_{F(P)} \rangle$ *and* $R_{F(P)}$ *is the mapping from* P *to* $F(P)$ *induced by* F*:*

(TRef1) Φ_P *is satisfiable* $\Rightarrow \Phi_{F(P)}$ *is satisfiable*
(TRef2) $\Phi_{F(P)} \Rightarrow R_{F(P)}(\Phi_P)$

These conditions mirror the conditions in Definition 5. Thus, the objective of the method we describe below is to determine whether a given *MAVO* transformation F is a refining transformation. Definition 5 for *MAVO* refinement assumes that both P and P' are over the same metamodel and we use the same restriction for the *MAVO* transformations we consider.

We further assume that a candidate refining transformation F is implemented as a set $\{\rho_1, ..., \rho_n\}$ of confluent and terminating *refinement rewrite rules*, defined as follows.

Definition 7 (*MAVO* refinement rewrite rule) *A* refinement rewrite rule ρ *on a* $MAVO(T)$ *model is a* MAVO *mapping* $R_\rho(LHS, RHS)$ *s.t.* LHS *and* RHS *are* $MAVO(T)$ *models. Rule* ρ *is applied to a* $MAVO(T)$ *model* P *by finding an occurrence of* LHS *in* P *and replacing it with* RHS *to produce* P'*. The resulting refinement mapping between* P' *and* P *consists of* R_ρ *at the site of the rule application and the identity mapping everywhere else.*

Default annotations are not used in a rewrite rule and so even ground annotations must be specified explicitly (i.e., E, P, C and COMP). In addition, we use the placeholder "?" when the annotation is not given. For example, in the rule for *ReduceAbs* (Figure 3), the element `E` on the LHS can match an element with any annotation as long as it includes S. Furthermore, if "?" is used in the same position on the RHS then it must represent the same value as the instantiation on the LHS. For example, if the element `E` on the LHS matches an element annotated with $\langle$M, S, C$\rangle$, then the RHS would be instantiated as $\langle$M, P, C$\rangle$ for element `E`.

In order to verify that a *MAVO* transformation is refining, we must prove that properties TRef1 and TRef2 in Definition 6 hold. To simplify this process, we note that the rules in our transformations are confluent (and thus, the order of their application does not matter) and that both TRef1 and TRef2 are transitive properties[1]. Thus, to verify each property it is sufficient just to check it on a single

[1] If transformations F and F' satisfy a transitive property then so does the composition $F \circ F'$.

rule application for each rewrite rule of the transformation. The above process is not necessary: even if verification of a particular rule application verification fails, the combined action of multiple ones rule may still be a refinement.

The applications of a rule $\rho = R_\rho(LHS, RHS)$ are exactly the simple extensions of it as specified in Definition 4. Thus, for example, to check TRef1 for ρ we must check that every simple extension $R(P_{LHS}, P_{RHS})$ of ρ satisfies Ref1.

We summarize the verification method as follows: Given a *MAVO* transformation F implemented as a set $\{\rho_1, ..., \rho_n\}$ of *MAVO* refinement rewrite rules, for each rule $\rho_i \in \{\rho_1, ..., \rho_n\}$, we must check that it satisfies TRef1 and TRef2. If these conditions hold for all rules ρ_i then F is a refining transformation. In the next two sections we present the method for checking TRef1 and TRef2 on a rule ρ_i by checking every simple extension $R(P_{LHS}, P_{RHS})$ of ρ_i.

4 Checking Property TRef1

To prove that TRef1 holds for a rule ρ requires showing that for each simple extension $R(P_{LHS}, P_{RHS})$ of ρ, if $FO(P_{LHS})$ is satisfiable then $FO(P_{RHS})$ is satisfiable as well. The proof of this property is dependent both on the metamodel constraints and the *MAVO* constraints. Our method requires the use of tool support for this step.

Specifically, we have developed tooling that, given ρ, produces an Alloy module that checks TRef1 in a bounded way by exploring all simple extensions $R(P_{LHS}, P_{RHS})$ of ρ up to a given scope. If Alloy finds that TRef1 does not hold for ρ, the counterexample it produces provides a way to "repair" the rule by adding a guard (e.g., a negative application condition [15]) that will prevent it from being applied in the bad cases. If Alloy reaches the scope without finding a counterexample, we cannot conclude that TRef1 holds. Thus this approach can only be used *to provide evidence* that TRef1 holds.

Our tool accepts a rule expressed in Ecore [30] as its input. The rule is then translated, using TXL [6], into an Alloy [18] encoding, which includes all of the rule's *MAVO* annotations, and is combined with our encoding of the Ecore metamodel. We encoded further Alloy predicates that allow us to create arbitrary simple extensions (see Definition 4) for the *LHS* and the *RHS* of the rule, as described in Section 3. The description corresponding to the extensions of the *LHS* of the rule was encoded using Alloy's `facts`, whereas that for the *RHS* – with Alloy's `pred`icates. This allowed us to only take into account well-formed *LHS* extensions, and to create instances of *RHS* extensions that are not well-formed, using Alloy's `assert`ions. Figure 7 shows an example snippet that demonstrates our Alloy encoding for rule *ReduceAbs*.

Running the generated Alloy encoding enumerates all concretizations of the *RHS MAVO* model (i.e., the *RHS* of the *MAVO* rule and its simple extensions) that are not well-formed, up to a given scope.

This process uses concretizations rather than *MAVO* models but produces the same result: assume that Alloy produces a concretization that is not well formed. That means that there exists a *ground MAVO* simple extension (i.e.,

```
sig CM in classLHS {}
sig CM' in classRHS {}
... // and other known signatures
sig classExtension in Class {}
... // and signatures for the simple extension
pred exists(in: set Class) { some in }
... //And other predicates for enforcing MAVO annotations
pred source{
    exists[CM] and
    ... //Other predicates for MAVO constraints on CM as appropriate, and for other
        //elements in the model
}
//And similarly for target, with CM' instead
fact {
    noDoubleOwner[compositionExtension+compositionLHS, classExtension+classLHS]
    ... //And other constraints that enforce the metamodel
}
pred {
    noDoubleOwner[compositionExtension+compositionRHS, classExtension+classRHS]
    ... //And other constraints that enforce the metamodel
}
assert refined { (target[] and source[]) implies consistentTarget[]  }
```

Fig. 7. Alloy encoding snippet.

containing only elements annotated with P, C, E) of the RHS of the $MAVO$ rule that has at least one concretization (the one that was found) that is not well-formed. Therefore, there exists an input $MAVO$ model for which the rule does not preserve consistency, and Alloy's enumeration process finds it.

Moreover, the above procedure does not miss any counterexamples up to the given scope. To ensure this, we constructed the $MAVO$ simple extensions of the rule with all elements having the "least restrictive" $MAVO$ annotations (S, V, M). In enumerating all concretizations of this $MAVO$ model, Alloy will end up enumerating the concretizations of all other (more restrictive) $MAVO$ simple extensions of the rule for the given scope. In other words, the set of concretizations of any $MAVO$ model (up to the given scope) is some subset of the set of concretizations of the "least restricted" $MAVO$ model, and therefore no counterexamples are overlooked.

5 Checking Property TRef2

Property Ref2 in Definition 5 has a desirable "locality" characteristic given by the following proposition.

Proposition 1 *Given a refinement rewrite rule $\rho = R_\rho(LHS, RHS)$, if ρ satisfies Ref2 then every simple extension $R(P_{LHS}, P_{RHS})$ of ρ satisfies Ref2.*

Thus, we can dramatically simplify the effort of proving condition TRef2 by simply checking Ref2 on the LHS and RHS of the rule itself. Furthermore, although Ref2 is general enough that it can be used with $MAVO$ models that are augmented with arbitrary FO constraints for expressing detailed cases of uncertainty, when we limit ourselves to just using $MAVO$ annotations, we can simplify checking the refinement condition Ref2 by defining syntactic constraints (i.e., sufficient conditions) on the annotations.

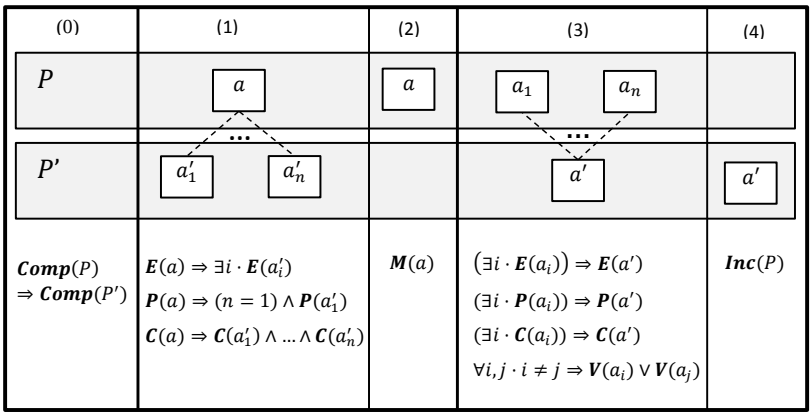


Fig. 8. Summary of the constraints on annotations of model elements across a *MAVO* refinement mapping.

Figure 8 summarizes these constraints, first introduced in [26], which we refer to as *MAVO syntactic refinement conditions*. Each of the five columns indicates a different case (case number is on the top) in the refinement mapping, and the sentences in the lower part of each case give the constraints on the *MAVO* annotations for the atoms of that case. A valid refinement must satisfy all of these constraints. The sentences make reference to the full set of *MAVO* annotations (M/E; S/P; V/C; INC /COMP), including those assumed by default when the annotation for a partiality type is omitted. specified by default by omitting the annotation for a partiality type). each non-ground/ground pair.

Case (0) says that if P is complete then P' must be as well. In case (1), when an atom a of model P is refined to a set of atoms $a_1, ..., a_n$ of P', the first sentence says that if a is annotated with E (i.e, it is not M), then at least one of the atoms a_i must also be annotated with E. Thus, if a exists and it is refined to the set of a_is then at least one of these should exist. The second sentence says that if a is a particular (i.e., *not* a set) then there can only be one a_i and it too must be a particular. The third sentence says that if a is a constant and thus it can't merge with any other atom then neither can any of the a_is it refines to and so they too must be constants. Case (2) says that if a is not propagated into the new model, then it must have been annotated with M. Case (3) states that if multiple a_is in P are mapped into a single a' in P', then if any of the a_is had definite information, or were particular, or were a constant, then so is a'. The last sentence in case (3) says that at most one of the a_is could be a variable. Finally, if a new atom, not mapped to anything in P, appears in P' (case (4)), then P could not be complete. For example, using this method it is clear that the refinement in Figure 1 is correct.

For a given refinement rewrite rule, the condition TRef2 can be easily checked with existing tools. In particular, checking TRef2 involves (a) expressing the constraints shown in Figure 8 as OCL constraints over the Ecore representation of

the rule, and (b) using an off-the-shelf OCL constraint checker, such as DresdenOCL [17].

5.1 Proof of Proposition 1

We prove this by using the syntactic refinement conditions as the definition for Ref2.

Proof. The proof is by induction on the number of atoms in the simple extension $R(P_{LHS}, P_{RHS})$ of ρ. The base case is ρ itself and it satisfies the syntactic refinement conditions by assumption. For the inductive step, we show that if the simple extension $R(P_{LHS}, P_{RHS})$ of ρ satisfies the syntactic refinement conditions then so does the simple extension $R^{\#}(P^{\#}_{LHS}, P^{\#}_{RHS})$ that is minimally larger. We construct $R^{\#}(P^{\#}_{LHS}, P^{\#}_{RHS})$ by choosing an atom α, $\alpha \notin P_{LHS}$, $\alpha \notin P_{RHS}$ and define $P^{\#}_{LHS} = P_{LHS} \cup \{\alpha\}$, $P^{\#}_{RHS} = P_{RHS} \cup \{\alpha\}$ and $R^{\#} = R \cup \{\langle \alpha, \alpha \rangle\}$. $R^{\#}(P^{\#}_{LHS}, P^{\#}_{RHS})$ is the unique (up to isomorphism) simple extension of $R(P_{LHS}, P_{RHS})$ with the least additional atoms. Although the atom α can have any annotation, we will initially consider the case that it is annotated with EPC in both $P^{\#}_{LHS}$ and $P^{\#}_{RHS}$.

To check whether $R^{\#}(P^{\#}_{LHS}, P^{\#}_{RHS})$ satisfies the syntactic refinement conditions, first note that since case (0) is not dependent on atoms, it must be satisfied by $R^{\#}(P^{\#}_{LHS}, P^{\#}_{RHS})$ since, by assumption, $R(P_{LHS}, P_{RHS})$ satisfies it. Next we must check the constraints in cases (1) and (2) for each atom a in $P^{\#}_{LHS}$. First consider the atom $a = \alpha$ in $P^{\#}_{LHS}$. It is mapped to a single atom α in $P^{\#}_{RHS}$ and so only case (1) applies and all the constraints are clearly met. Every other atom $a \neq \alpha$ in $P^{\#}_{LHS}$ is also in P_{LHS} and so, by the inductive assumption and the fact that α is not mapped to any of these, we can conclude that cases (1) and (2) are satisfied for these. We can argue similarly, for the atoms of $P^{\#}_{RHS}$ and show that all the constraints for cases (3) and (4) are met.

Therefore, when α is annotated with EPC, $R^{\#}(P^{\#}_{LHS}, P^{\#}_{RHS})$ satisfies all the syntactic refinement conditions. Now, if any of these annotations are weakened the result is that fewer syntactic refinement conditions are applicable but this does not change the fact that they are all satisfied. For example, if α in $P^{\#}_{LHS}$ is annotated with MPC, then it must have the same annotation in $P^{\#}_{RHS}$ (by definition of a simple extension), and the first constraint in cases (1) and (3) no longer applies.

Therefore, for any annotation on α, $R^{\#}(P^{\#}_{LHS}, P^{\#}_{RHS})$ satisfies all the syntactic refinement constraints. □

6 Applying the Verification Method

We now illustrate the verification method on three transformations of *MAVO* partial models defined by rewrite rules. In all cases, the transformation is obtained by applying the corresponding rule(s) repeatedly until it can no longer

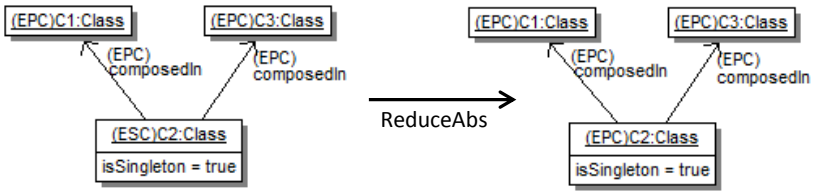


Fig. 9. An example of *ReduceAbs* producing an inconsistent model.

be applied. The result of this process is unique regardless of the order of rule applications due to the assumption of confluence and is guaranteed to terminate due to the assumption of termination. We show how results of the analysis can either give evidence of correctness of each transformation or help repair it.

Example Transformation Rules. The first example is the language-independent transformation *ReduceAbs* discussed in Section 1 with the rule shown in Figure 3. The second is *GetSet* with the rule shown in Figure 10(a). *GetSet* is a simple detail-adding refinement transformation for class diagrams that we "lift" so that it can be applied to *MAVO* class diagrams. Our objective here is to examine the common situation where partiality-reducing refinements are interleaved with detail-adding ones. *ReduceAbs* and *GetSet* are toy transformations and are considered here because they have been analyzed manually in [25], whereas here we show how our method can automate it.

The third example is *CompReduce*, shown in Figure 10(b). It consists of four rules. This transformation has pragmatic utility: there are cases in *MAVO* models when a refinement can be *implied* by the interaction between annotations and well-formedness rules, and `CompReduce` constructs these implied refinements for instances of the `composedIn` association. Rule (R1) encodes the fact that if an instance of the `composedIn` relation exists (i.e., is E-annotated) between two classes then the classes must exist as well, since an association cannot exist without its endpoints. Rule (R2) is due to the the well-formedness constraint in Figure 5 that forbids a singleton class from being composed in two classes simultaneously. The rule says that the S-annotated class `C2` on the LHS can be split since it has two EC-annotated `composedIn` associations and thus any concretization of `C2` must have at least two classes. Rule (R3) says that the `composedIn` association between two P-annotated classes can only be particular (i.e., there cannot be a set of them) and so it should be P-annotated as well. Finally, rule (R4) is similar to (R3) but for C-annotated classes.

Verifying the transformations. Table 1 shows the results of applying the method to the six rules of the three example transformations. The experiments were run on a laptop with an Intel Core i7 processor and 8 GB of RAM using Alloy 4.2. For each rule, we report the results of checking TRef1 and TRef2. For rules that fail TRef1, we record the minimum scope for a failure. For those that pass it, we record the maximum scope that could be checked before the solver ran out of memory. We also report the overall time required for Alloy to create the first counterexample, if any.

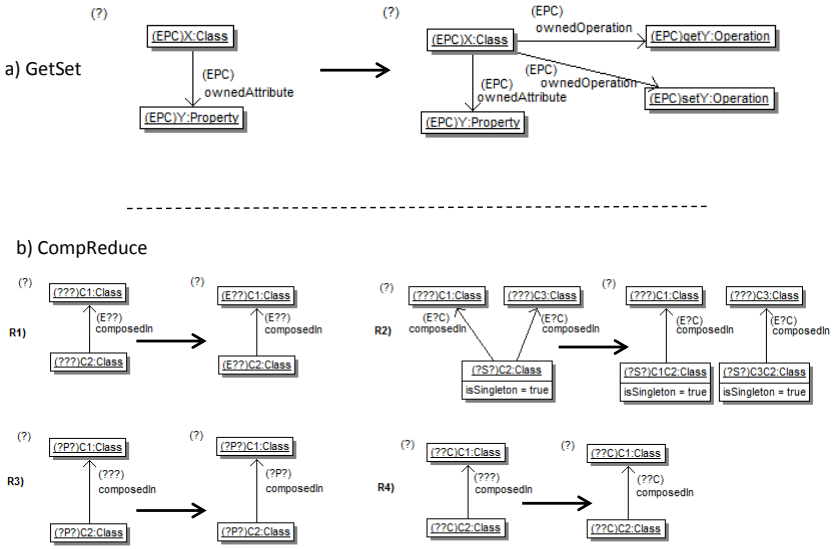


Fig. 10. The rules defining transformations *GetSet* and *CompReduce*.

TRef2 holds for *ReduceAbs* but TRef1 fails to hold and a counterexample is shown in Figure 9. The LHS is a model with an S-annotated singleton class `C2` that has two `composedIn` associations to different classes. The RHS changes `C2` to being P-annotated by applying *ReduceAbs*. The LHS has concretizations but the RHS does not because of the well-formedness constraint in Figure 5 that forbids a singleton class from being composed into two classes simultaneously.

One way to repair this rule is to restrict it by adding a negative application condition (NAC) [15] that guards the rule application from situations such as this. The NAC is created by encoding the relevant slice of the discovered counterexample. The resulting fixed rule satisfies both Tref1 (up to scope 20) and Tref2, as shown in Table 1.

Another interesting way to repair *ReduceAbs* is to *restrict* it to apply only after the *CompReduce* transformation, to "normalize" the input model. In this case, rule (R2) of *CompReduce* would split the problematic case into two S-annotated singleton classes and then *ReduceAbs* could be applied. Note that these two possible repairs yield different results.

TRef1 holds for the scope 20 for the transformation `GetSet`, but TRef2 fails. The counterexample here occurs when the RHS model is COMP -annotated since the elements `getY : Operation`, `setY : Operation` and the corresponding `ownedOperation` relations are added to the LHS but case (4) in Figure 8 says that such additions can only occur in a refinement if the model is INC -annotated (i.e., incomplete). Thus, we repair this rule by refining the *OW* annotation from ? to INC . The resulting transformation satisfies both properties.

For the four rules of the transformation *CompReduce*, both TRef2 and TRef1 are satisfied and thus we have evidence that this transformation is uncertainty-reducing.

Rule	TRef1	Scope	Time (ms)	TRef2
ReduceAbs	fail	3	562	pass
ReduceAbs(repaired)	pass	20	10593296	pass
GetSet	pass	15	925032	fail
GetSet (repaired)	pass	20	10706024	pass
CompReduce (R1)	pass	9	2502127	pass
CompReduce (R2)	pass	15	3292782	pass
CompReduce (R3)	pass	10	1035124	pass
CompReduce (R4)	pass	10	3901705	pass

Table 1. Results of applying the verification method to the six rules and their counter-example-based repairs.

7 Related Work

The uncertainty-reducing transformations that we studied in this paper are closely related to refinement of partial behavioral models. Well known examples of such formalisms include Modal Transition Systems (MTSs) [21] and Featured Transition Systems (FTSs) [5]. The concretizations of MTSs and FTSs are Labeled Transition Systems (LTSs).

In MTSs, uncertainty is captured using *maybe*-annotated transitions) Existing methods of checking MTS refinement, e.g., [22,13], verify that it holds for specific pairs of models. Our approach, on the other hand, aims to verify that a transformation is refining regardless of particular input and output models.

Featured Transition Systems (FTSs) [5] are *precise* representations of sets of models, used in the area of Software Product Line (SPL) engineering [23] to capture the variability in the behavior of products in a product family. An FTS encodes a set of LTSs using annotations that associate each of its transitions with specific features from a feature diagram. FTS refinement is studied in [7] for the case where new features are added to the SPL, by classifying the new features w.r.t. whether they add or remove new behavior. MAVO partiality can express more nuanced kinds of variability than the M-like variability in FTSs. This impacts the difficulty of verification of refinement of MAVO models. Moreover, the notion of feature-set evolution for which FTSs refinement is studied, means that at a certain level, verification also happens at the level of specific pairs of partial models. Finally, MAVO refining transformations can describe more general scenarios where uncertainty is systematically removed from a system.

More broadly, our work is related to a number of approaches for verifying properties of model transformations. Some of them employ theorem proving [14,28], whereas others do some form of model checking [16,4]. Like our approach to proving TRef1, many use Alloy. For example, Baresi et al. [2] represent subsequent applications of rules to an input model as a state-space, similarly to the standard method for representing traces with Alloy [19]. This allows property checking for graph transformation systems, similar to bounded model checking. Anastasakis et al. [1] take a similar approach, using Alloy to verify ATL-like transformations [20]. They create the Alloy encoding of the transformation and

its source and target metamodels and run the tool to produce instances of transformed models, trying to verify that, given well-formed inputs, the rule produces well-formed outputs. However, neither of the above approaches proposes a systematic method to repair the transformation in case counter-examples are produced. Sen et al. [29] use Alloy to create complete versions of partially defined models to use for *testing* model transformations. This process is reminiscent of the way we use Alloy to generate all extensions of the graph rewrite rule, even though the eventual goal is different.

8 Conclusion and Future Work

This paper presented a contribution to model-driven engineering in the context of uncertainty. Specifically, we described an automated approach to showing correctness of uncertainty-reducing transformations on partial models. The approach builds on our earlier work on verifying uncertainty-reducing *refinement applications* [25]. We extended the two proof obligations from [25] to showing that they hold for all possible input/output pairs of models. We then showed that the first condition can be checked using a special-purpose tool built on top of Alloy, and the second condition can be checked using a standard OCL checker using a set of syntactic conditions on the transformation. Applying the method on several examples showed that it is effective for debugging transformations and for gathering evidence of their correctness.

Our approach has a number of limitations which we intend to address in later work. Specifically, we are interested in investigating ways to *prove* the transformation condition TRef1, instead of collecting evidence for it using Alloy. In some cases, this can be done by calculating the maximum scope under which an absence of a counterexample guarantees correctness. This notion is similar to a problem *diameter* [3]. We also plan to study the more general problem of verifying uncertainty-reducing refining transformations that also involve metamodel translations.

References

1. K. Anastasakis, B. Bordbar, and J. Küster. Analysis of Model Transformations via Alloy. In *Proc. of MoDeVVa'07*, 2007.
2. L. Baresi and P. Spoletini. On the Use of Alloy to Analyze Graph Transformation Systems. In *Proc. of ICGT'06*, pages 306–320, 2006.
3. A. Biere, A. Cimatti, E. M. Clarke, and Y. Zhu. Symbolic Model Checking without BDDs. In *Tools and Algorithms for the Analysis and Construction of Systems (TACAS'99)*, volume 1579 of *LNCS*. Springer, 1999.
4. A. Boronat, R. Heckel, and J. Meseguer. Rewriting Logic Semantics and Verification of Model Transformations. In *Proc. of FASE'09*, volume 5503 of *LNCS*, pages 18–33, 2009.
5. P. Classen, A.and Heymans, P.Y. Schobbens, A. Legay, and J.F. Raskin. Model Checking Lots of Systems: Efficient Verification of Temporal Properties in Software Product Lines. In *Proc. of ICSE'10*, pages 335–344, 2010.
6. J. Cordy. The TXL Source Transformation Language. *Sci. Comput. Program.*, 61(3):190–210, 2006.
7. M. Cordy, A. Classen, P.Y. Schobbens, P. Heymans, and A. Legay. Managing Evolution in Software Product Lines: a Model-checking Perspective. In *Proc. of VaMoS'12*, pages 183–191, 2012.
8. C Ebert and J De Man. Requirements Uncertainty: Influencing Factors and Concrete Improvements. In *Proc. of ICSE'05*, pages 553–560, 2005.
9. H. Ehrig, K. Ehrig, U. Prange, and G. Taentzer. *Fundamentals of Algebraic Graph Transformation (Monographs in Theoretical Computer Science. An EATCS Series)*. Springer, 1 edition, 2006.
10. M. Famelis, S. Ben-David, M. Chechik, and R. Salay. Partial Models: A Position Paper. In *Proceedings of MoDeVVa'11*, pages 1–6, 2011.
11. M. Famelis, M. Chechik, and R. Salay. Partial Models: Towards Modeling and Reasoning with Uncertainty. In *Proc. of ICSE'12*, 2012. To appear.
12. M. Famelis, R. Salay, and M. Chechik. The Semantics of Partial Model Transformations. In *Proc. of MiSE'12*, 2012. To appear.
13. D. Fischbein, G. Brunet, N. D'Ippolito, M. Chechik, and S. Uchitel. Weak Alphabet Merging of Partial Behaviour Models. *ACM TOSEM*, 21(2):1–49, 2011.
14. H. Giese, S. Glesner, J. Leitner, W. Schäfer, and R. Wagner. Towards Verified Model Transformations. In *Proc. of MoDeVVa'06*, pages 78–93, 2006.
15. A. Habel, R. Heckel, and G. Taentzer. Graph Grammars with Negative Application Conditions. *Fundamenta Informaticae*, 26(3):287–313, 1996.
16. R. Heckel. Compositional Verification of Reactive Systems Specified by Graph Transformation. In *Proc. of FASE'98*, pages 138–153, 1998.
17. H. Hussmann, B. Demuth, and F. Finger. Modular Architecture for a Toolset Supporting OCL. In *Proc. of UML'00*, volume 1939 of *LNCS*, pages 278–293, 2000.
18. D. Jackson. *Software Abstractions: Logic, Language, and Analysis*. MIT Press, 2006.
19. D. Jackson, I. Shlyakhter, and M. Sridharan. A Micromodularity Mechanism. In *Proc. of FSE'01*, pages 62–73, 2001.
20. F. Jouault, F. Allilaire, J. Bézivin, and I. Kurtev. ATL: A Model Transformation Tool. *Science of Computer Programming*, 72(1):31–39, 2008.
21. K. G. Larsen and B. Thomsen. A Modal Process Logic. In *Proc. of LICS'88*, pages 203–210, 1988.

22. P. Larsen. The Expressive Power of Implicit Specifications. In *Proc. of ICALP'91*, volume 510 of *LNCS*, pages 204–216, 1991.
23. K. Pohl, G. Böckle, and F. Van Der Linden. *Software Product Line Engineering: Foundations, Principles, and Techniques.* Springer-Verlag New York Inc, 2005.
24. M. Sabetzadeh, S. Nejati, M. Chechik, and S. Easterbrook. Reasoning about Consistency in Model Merging. In *Proc. of LWI'10*, 2010.
25. R. Salay, M. Chechik, and J. Gorzny. Towards a Methodology for Verifying Partial Model Refinements. In *Proc. of VOLT'12*, April 2012. to appear.
26. R. Salay, M. Chechik, and J. Horkoff. Managing Requirements Uncertainty with Partial Models, March 2012. submitted.
27. R. Salay, M. Famelis, and M. Chechik. Language Independent Refinement Using Partial Modeling. In *Proc. of FASE'12*, volume 7212 of *LNCS*, pages 224–239, March 2012.
28. B. Schätz. Verification of Model Transformations. *ECEASST*, 29, 2010.
29. S. Sen, J.M. Mottu, M. Tisi, and J. Cabot. Using Models of Partial Knowledge to Test Model Transformations. In *Proc. of ICMT'12'*, 2012.
30. D. Steinberg, F. Budinsky, M. Paternostro, and E. Merks. *EMF: Eclipse Modeling Framework.* Addison Wesley, 2007.
31. A. van Lamsweerde. *Requirements Engineering - From System Goals to UML Models to Software Specifications.* Wiley, 2009.