CSC412 / CSC2506 Sample Problems for Midterm

1. Let $p(k)$ be a one-dimensional discrete distribution that we wish to approximate, with support on non-negative integers. One way to fit an approximating distribution $q(k)$ is to minimize the Kullback-Leibler divergence:

$$KL(p||q) = \sum_{k=0}^{\infty} p(k) \log \frac{p(k)}{q(k)}$$

Show that when $q(k)$ is a Poisson distribution,

$$q(k) = \frac{\lambda^k \exp(-\lambda)}{k!}$$

this KL divergence is minimized by setting $\lambda$ to the mean of $p(k)$.

2. Recall that the definition of an exponential family model is:

$$f(x|\eta) = h(x) g(\eta) \exp(\eta^T T(x))$$

where:

- $\eta$ are the parameters
- $T(x)$ are the sufficient statistics
- $h(x)$ is the base measure
- $g(\eta)$ is the normalizing constant

Consider the univariate Gaussian, with mean $\mu$ and precision $\lambda = \frac{1}{\sigma^2}$:

$$p(D|\mu, \lambda) = \prod_{i=1}^{N} \left(\frac{\lambda}{2\pi}\right)^{\frac{1}{2}} \exp\left(-\frac{1}{2}(x_i - \mu)^2\right)$$

What are $\eta$ and $T(x)$ for this distribution when represented in exponential family form?
3. Consider the following directed graphical model:

(a) List all variables that are independent of $A$ given evidence on $B$.
(b) Write down the factorized normalized joint distribution that this graphical model represents.

4. Murphy 20.1

Correction! Older copies of the Murphy book have a typo pointing you to an incorrect figure. Do this question but with this MRF:

5. Consider the Factor Graph:

(a) Write down the normalized joint distribution $P(X_1, X_2, X_3, X_4, X_5)$ in terms of the potentials.
(b) Write down any conditional independence relationships given by the graph.