

Tutorial Exercise 6: Catch Up and Network Flow

1. **Reading Week.** Catch up on your studying for the course and bring your unresolved questions to the next tutorial, which is on Mon., Feb 26.
2. **Teaching Assignments.** You are chair of the Department of Computer Science, and are given the following information. There is a set of profs $P = \{p_n\}_{n=1}^N$, a set of courses $C = \{c_k\}_{k=1}^K$, and the number of course sections $S = \{s_k\}_{k=1}^K$ being offered next term. Here s_k is a positive integer equal to the number of different sections of course c_k that need to be taught next term. In addition, for each prof p_n , you are given the set of courses that he/she likes to teach, say $T_n \subset C$, along with her teaching load L_n . This teaching load L_n is the number of courses that prof p_n is assigned to teach next term (here $L_n \geq 0$ is an integer). Finally, suppose the sum of teaching loads, $L = \sum_{n=1}^N L_n$, is equal to the number of course sections that need to be taught, $S = \sum_{k=1}^K s_k$.

The prof-to-course assignment problem is then to assign each prof p_n to exactly L_n course sections, with the courses themselves chosen from T_n , and ensure that each section of each course gets assigned a professor.

- (a) Represent the prof-to-course assignment problem as a $s - t$ network flow problem. Describe exactly the relationship between flows in your graph and prof-to-course assignments. That is, on one hand, given any flow (or perhaps a restricted family of flows) explain what assignment it represents. And, on the other hand, given an assignment explain how this assignment can be represented as a valid flow in your graph.
- (b) One question we might ask is whether an assignment is even possible? Can the professors cover all the sections of all the courses? Describe in detail how you can answer that question using your network flow. (You can use algorithms described in the course notes or text without writing them out again.)
- (c) In situations where a prof-to-course assignment is possible, explain how your network flow provides a suitable assignment. In particular, explain why this assignment must satisfy the constraints of the prof-to-course assignment problem.
- (d) In situations where it is not possible to cover all the courses, how can you clearly convince the faculty that no such assignment is possible? Your argument must be worded in terms of the sets of courses professors like to teach, their teaching loads, and the sections that need to be taught, and **not** in terms for your network flow problem. It must be verifiable from these given quantities using only simple arithmetic.