Question 5. (Properties of Network Flow) [15 MARKS]

Suppose we have a network $N = (V, E)$ with a maximum flow f on N, with the property that there is **no** cycle in N on which all of the edges have non-zero flow (*i.e.*, every cycle in N contains at least one edge e for which $f(e) = 0$). Let $e \in E$ be an edge used at maximum capacity ($f(e) = c(e)$) and suppose the capacity of e is reduced to $c'(e) < c(e)$. Then, the maximum flow f must be adjusted to f' so that $f'(e) \leqslant c'(e)$, while preserving the properties of a valid flow.

Give an efficient algorithm that takes N, f, and e and that determines a maximum flow f' in the network N', where N' is the same as N except for $c'(e) = c(e) - 1$. Include a brief English description of the main idea of your algorithm, justify that your algorithm is correct, and analyze its running time.