Question 4. (Dynamic Programming) [20 marks]

You are running a software business with client base in Toronto and Vancouver. During every month you are offered one project in each of these cities. Each project has a different payoff. We denote the payoff during week i for the Toronto project by \( T_i \) and for the Vancouver project by \( V_i \). You must be present in a city to undertake a project there. Every time you move you incur a cost of \( C \) units. We assume that on week 0 you are located in Toronto.

In addition to the two projects mentioned above, every month you have an opportunity to work on a project over the internet, with a payoff of \( I_i \). This project has the advantage that it can be done anywhere, and does not require you to move.

For example, if the payoff table is

\[
\begin{array}{c|cccccc}
\text{week} & 1 & 2 & 3 & 4 & 5 \\
\hline
I_i & 50 & 50 & 30 & 50 & 10 \\
T_i & 100 & 100 & 40 & 70 & 10 \\
V_i & 100 & 50 & 100 & 10 & 100 \\
\end{array}
\]

and \( C = 50 \) then the optimal sequence of projects is \( T, T, V, I, V \). This requires one move and achieves a total profit of \( 100 + 100 + 100 + 50 + 100 - 50 = 400 \).

Give an efficient algorithm that given a table of \( I_i, T_i \) and \( V_i \) for the first \( n \) weeks outputs the profit of the optimal sequence of projects. You do not need to compute the actual sequence.

**Hint:** One possible solution uses a \( 2 \times (n + 1) \) array.