Question 4. (Dynamic Programming) [20 MARKS]

You are running a software business with client base in Toronto and Vancouver. During every month you are offered one project in each of these cities. Each project has a different payoff. We denote the payoff during week i for the Toronto project by T_i and for the Vancouver project by V_i. You must be present in a city to undertake a project there. Every time you move you incur a cost of C units. We assume that on week 0 you are located in Toronto.

In addition to the two projects mentioned above, every month you have an opportunity to work on a project over the internet, with a payoff of I_i. This project has the advantage that it can be done anywhere, and does not require you to move.

For example, if the payoff table is

week	1	2	3	4	5
I_i	50	50	30	50	10
T_i	100	100	40	70	10
V_i	100	50	100	10	100

and $C = 50$ then the optimal sequence of projects is T, T, V, I, V. This requires one move and achieves a total profit of $100 + 100 + 100 + 50 + 100 - 50 = 400$.

Give an efficient algorithm that given a table of I_i, T_i and V_i for the first n weeks outputs the *profit* of the optimal sequence of projects. You do not need to compute the actual sequence.

Hint: One possible solution uses a $2 \times (n + 1)$ array.