

PLEASE HAND IN

UNIVERSITY OF TORONTO  
Faculty of Arts and Science  
DECEMBER EXAMINATIONS 2006

PLEASE HAND IN

CSC 487/2503H F  
St. George Campus  
Duration — 3 hours  
Aids allowed: none

Student Number:

Family Name:

Given Name:

*Do not turn this page until you have received the signal to start.*  
(In the meantime, please fill out the identification section above,  
and read the instructions below.)

This examination consists of 6 questions on 14 pages (including this one). *When you receive the signal to start, please make sure that your copy of the examination is complete.* If you need more space for one of your solutions, use the reverse side of the page and *indicate clearly the part of your work that should be marked.* Also, the last two pages are provided for scratch work.

Write your student number at the bottom of pages 2-14 of this test.

# 1: \_\_\_\_\_/19

# 2: \_\_\_\_\_/15

# 3: \_\_\_\_\_/16

# 4: \_\_\_\_\_/16

# 5: \_\_\_\_\_/12

# 6: \_\_\_\_\_/10

TOTAL: \_\_\_\_\_/88

*Good Luck!*

**Question 1.** [19 MARKS]

Give a short answer for each of the following questions.

**Part (a)** [3 MARKS] In terms of the three objective criteria used by Canny to find an optimal edge detector, describe in words which ones the square wave filter (depicted below) optimizes?



What problems might one encounter when trying to use such a filter for edge detection?

**Part (b)** [3 MARKS] Say you want a feature point detector for detecting long range motion and recovering camera and scene geometry. Give three key properties that it should have (i.e., what makes a good feature detector)?

**Part (c)** [4 MARKS] What special difficulties would you expect to have in computing optical flow given spatially aliased images? What special difficulties would you expect to have in computing optical flow given a temporally aliased image sequence? What can you do to deal with these situations?

**Part (d)** [3 MARKS] State the NCut objective function mathematically and describe the intuition behind it.

**Part (e)** [6 MARKS] Suppose one has a state space representation of the dynamical system with the state at time  $t$  given by  $\vec{x}_t$  and the state history by  $\vec{x}_{0:t}$ . Let the corresponding observation history be denoted by  $\vec{z}_{1:t}$ . What is the *filtering distribution*, and what is its relation to the posterior distribution  $p(\vec{x}_t | \vec{z}_{1:t})$ ? Give a recursive expression for the filtering distribution, and specify any assumptions you might exploit in order to derive this expression.

**Question 2.** [15 MARKS]

**Part (a)** [2 MARKS] Define the bidirectional reflectance distribution function (BRDF) in terms of radiometric quantities, also specifying the units in which these quantities are usually measured.

**Part (b)** [2 MARKS] State the mathematical form of the BRDF for a Lambertian surface?

**Part (c)** [2 MARKS] Give a mathematical expression for the radiance of a Lambertian surface illuminated with a distant point light source. Define clearly all elements of the expression.

**Part (d)** [3 MARKS] In simple photometric stereo algorithms, like that used in the first assignment, one often assumes Lambertian reflectance. Why is this assumption convenient? What computationally useful property is lost if we were to use the Phong model instead?

**Part (e)** [6 MARKS] Suppose you wished to apply photometric stereo to objects with non-Lambertian reflectance, and you do not know the light source locations. But suppose that you have a reference object with known geometry (e.g., a sphere) that has the same material reflectance properties (i.e., the same BRDF), and that you can take multiple images of the objects (including the reference object) from the same viewpoint and with the same set of illuminants. Explain how you might solve this variation on the photometric stereo problem to find the surface shape of the test objects.

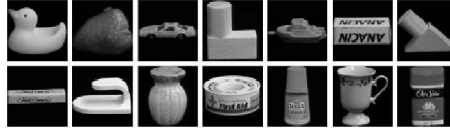
**Question 3.** [16 MARKS]

Suppose you are given an ensemble of images of an object for which you wish to learn a view-based appearance model. Assume there are  $N$  images denoted  $\{I_j(\vec{x})\}_{j=1}^N$ .

**Part (a)** [4 MARKS] State and briefly explain what steps you might take to learn such a view-based model using PCA.

**Part (b)** [5 MARKS] Given a subspace model learned with PCA, explain how you might build a simple detector to find instances of this object in an image.

**Part (c)** [4 MARKS] Murase and Nayar (1996) learned PCA models from image of objects like those shown below under variations in camera pose and light source position.



Explain what problems you detector might encounter with such objects and training images.

**Part (d)** [3 MARKS] Consider a planar surface with albedo variations (texture) and Lambertian reflectance. Suppose that it can be illuminated by various distant point light sources, but it is always viewed from one particular camera angle. Can you say something about the maximum number of basis images that you might need to represent the appearance variations of the object under all possible illuminants?

**Question 4.** [16 MARKS]

Suppose you are given 20 image points,  $\vec{x}_j = (x_j, y_j)$  for  $j = 1 \dots 20$ , to which you wish to fit an ellipse. As you know, one can express the implicit form of an ellipse as

$$(x^2, xy, y^2, x, y, 1) \cdot \vec{a} = 0 \quad (1)$$

where  $\vec{a} = (a_1, a_2, a_3, a_4, a_5, a_6)^T$  are the polynomial coefficients of the ellipse.

**Part (a)** [7 MARKS] Suppose the noise in the point measurements  $\vec{x}_j$  has heavy tails, and we therefore expect outliers. Describe a simple RANSAC algorithm to solve for the coefficients  $\vec{a}$ .

**Part (b)** [5 MARKS] Suppose noise in the measurements  $\vec{x}_j$  is Gaussian with mean 0 and covariance  $C_j$ . State the Gold Standard objective function that you should use to find the optimal estimate of the coefficients  $\vec{a}$ .

**Part (c)** [4 MARKS] The linear constraints in Part 1 of this question lead naturally to a least-squares formulation. Would the solution be the Gold Standard optimum? Why or why not?

**Question 5.** [12 MARKS]

Suppose one knows a pair of corresponding points in two images of a 3D scene. That is, let two 2D points in the left image be  $\vec{x}_1^L$  and  $\vec{x}_2^L$ , and assume that their corresponding points in the right image are  $\vec{x}_1^R$  and  $\vec{x}_2^R$ . Furthermore, suppose that the two epipolar lines associated with  $\vec{x}_1^L$  and  $\vec{x}_2^L$ , denoted by  $e(\vec{x}_1^L)$  and  $e(\vec{x}_2^L)$ , are the same line.

**Part (a)** [4 MARKS] Consider an additional point  $\vec{x}_0^L$  in the left image which corresponds to a point  $\vec{x}_0^R$  on the line  $e(\vec{x}_1^L)$ . What can you say about its location in the left image?

**Part (b)** [4 MARKS] Given a second pair of corresponding points  $\vec{x}_3^L$  and  $\vec{x}_4^L$  with identical epipolar lines,  $e(\vec{x}_3^L) = e(\vec{x}_4^L)$ , with  $e(\vec{x}_3^L) \neq e(\vec{x}_1^L)$ , where are the left and right epipoles?

**Part (c)** [4 MARKS] Using the four points  $\vec{x}_j^L$ ,  $j = 1..4$ , and their associated epipolar lines, show how you might solve for the fundamental matrix  $F$ ? [Hint: what is the relationship between epipolar lines and the fundamental matrix?]

**Question 6.** [10 MARKS]

Let  $A$  be a  $n \times m$  matrix. Suppose  $A$  is known to be a rank  $r$  matrix ( $r < \min(n, m)$ ) plus normally distributed additive noise:

$$A = LW + E \quad (2)$$

where  $L$  is  $n \times r$ , and  $W$  is  $r \times m$  and  $E$  is  $n \times m$ .

**Part (a)** [5 MARKS] Suppose  $r < \min(n, m)$ . Give an algorithm for recovering information about  $L$  and  $W$ . You will not be able to solve uniquely for  $L$  and  $W$ . Specify as precisely as you can the family of possible solutions.

**Part (b)** [5 MARKS] If we also know that particular rows of  $L$  are mutually orthogonal, say rows  $r_k$  and  $s_k$  for  $k = 1 \dots K$ , can we then determine  $L$  and  $W$  uniquely? What's the minimum number of pairs  $r_k, s_k$  you would need to reduce the family of possible solutions to be as small as possible? In doing so, specify as precisely as you can the remaining family of possible solutions.

For scratch work.

For scratch work.

Total Marks = 88

Student #: \_\_\_\_\_

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END OF EXAMINATION