Question 1. [14 MARKS]

For each of the following operations briefly describe the most efficient (in the big-O sense) algorithm. Give the worst-case runtime efficiency in big-O (using the smallest, simplest expression).

Part (a) [2 MARKS] Given a balanced binary search tree of n nodes containing integers, return true if it contains the number 32.

Algorithm: Binary search

Runtime efficiency: $O(\log(n))$

Part (b) [2 MARKS] Given a linked list of n nodes containing integers, move the node with the largest value to the end of the list.

Algorithm: Loop through list to find previous node to largest item. Move largest item.

Runtime efficiency: $O(n)$

Part (c) [2 MARKS] Given two arrays of n integers, check if every integer in the first array also appears in the second array.

Algorithm: Sort both arrays, then use an algorithm similar to merge.

Runtime efficiency: $O(n \log(n))$

Part (d) [2 MARKS] Given a balanced binary search tree of n nodes containing integers, return the biggest difference between any node and its successor.

Algorithm: Do an in-order traversal, keeping track of the biggest gap so far.

Runtime efficiency: $O(n)$

Part (e) [2 MARKS] Given a sorted array of n integers, return the smallest integer in the array which is larger than 42 (if any).

Algorithm: Do a binary search for 42, remember the last index checked. The value to be returned is either at that index or at the next index (if any).

Runtime efficiency: $O(\log(n))$

Part (f) [2 MARKS] Given an array of n integers, determine if there are two elements in this array which sum to 100.

Algorithm: Sort the array $O(n \log(n))$. For each item, use binary search to search for 100 - item.

Runtime efficiency: $O(n \log(n))$

Part (g) [2 MARKS] Given an array of n integers, check if there is a subset of this array whose elements sum to 0.

Algorithm: Recursively generate the list of sums of all subsets of the first k items. This list has length $\leq 2^k$. To process the $(k+1)^{st}$ item, append the sum of the $(k+1)^{st}$ item with each of the previous sums to the list (runtime $O(2^k)$). Return true if 0 ever appears as a sum.

Runtime efficiency: $O(2^n)$

Question 2. [14 MARKS]

Complete the method `has100Run` below. The only objects you may create are instances of the class `CircularQueue` from the lectures. This class has the constructor `CircularQueue(int capacity)` and implements the following interface:

```
public interface Queue {
    void enqueue(Object o);
    Object head();
    Object dequeue();
    int size();
    int capacity();
}
```

Recall the Integer class has a method `int intValue()` which returns the value. Also recall the `Iterator` interface has the methods:

```
public interface Iterator {
    Object next();
    boolean hasNext();
}
```

```
/** Returns whether i has a contiguous (i.e. with no gaps) subsequence of elements
 *  whose values add to exactly 100.
 *  Precondition: i produces only Integer objects, with strictly positive values. */
public static boolean has100Run(Iterator i)

    int sum = 0;
    Queue q = new CircularQueue(100);

    while (i.hasNext()) {
      // Loop Invariant: sum == sum of elements in queue.
      while(i.hasNext() && sum < 100) {
        int val = (Integer) i.next();  // uses auto-unboxing
        q.enqueue(val);  // uses auto-boxing
        sum += val;
      }
      while (sum > 100) {
        // q cannot be empty when sum > 0.
        int val = (Integer) q.dequeue(); // uses auto-unboxing
        sum -= val;
      }
      if (sum == 100) {
        return true;
      }
    }

    return false;
  }
```

Question 3. [12 MARKS]

Consider the following recursive method:

```
public static int m(int a, int b, int c) {
  if (a > b) {
    return m(b, a, c);
  } else if (b > c) {
    return m(a, c, b);
  } else {
    return c;
  }
}
```

Part (a) [4 MARKS]

Give a good Javadoc method comment for `m`.

```
/** Returns the maximum of the three integer arguments.
 *  @param a first integer
 *  @param b second integer
 *  @param c third integer
 *  @return the maximum of the three integers. */
```

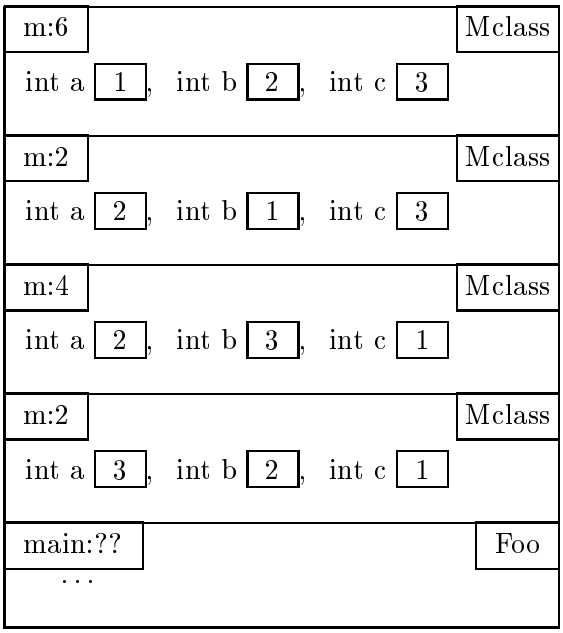
Part (b) [2 MARKS]

Give an example of concrete values `a`, `b` and `c` for which `m(a, b, c)` results in as deep a recursion as possible.

Answer: $(a, b, c) = (3, 2, 1)$

Part (c) [6 MARKS]

Show the runtime stack after calling `m` with your values of `a`, `b` and `c`, just before `return c` executes for the first time.



Question 4. [14 MARKS]

Consider the following class for nodes in a Linked List:

```
class ListNode {
  public Comparable data;
  public ListNode link;
}
```

Complete the following method. You **must not** use recursion. You can use non-recursive helper methods. Do not use any other datatypes such as arrays.

Recall that the `Comparable` interface consists of the method `int compareTo(Object o)`.

```
/** Merge two linked lists that are sorted in non-decreasing order into one
 *  sorted list containing all of the elements in the two given lists,
 *  including any duplicate elements.  Returns the head of the merged list.
 *  This method reuses the nodes in the original two lists.
 *  @throws java.lang.IllegalArgumentException (a RuntimeException) if
 *          either argument list h1 or h2 is not sorted.  */
public static ListNode merge(ListNode h1, ListNode h2) {

  ListNode prev = null;
  ListNode head = null;
  while (h1 != null && h2 != null) {
    if (h1.data.compareTo(h2.data) <= 0) {
      if (prev == null) {
        head = h1;
      } else {
        if (prev.data.compareTo(h1.data) > 0)
          throw new IllegalArgumentException("Argument list not sorted");
        prev.link = h1;
      }
      prev = h1;
      h1 = h1.link;
    } else {
      if (prev == null) {
        head = h2;
      } else {
        if (prev.data.compareTo(h2.data) > 0)
          throw new IllegalArgumentException("Argument list not sorted");
        prev.link = h2;
      }
      prev = h2;
      h2 = h2.link;
    }
  }
  // At least one of h1 and h2 is null.
  // Continued on next page...
```

```
  // The following does not check the remaining list is sorted.
  if (h1 != null) {
    if (head == null)
      head = h1;
    else
      prev.link = h1;
  }
  if (h2 != null) {
    if (head == null)
      head = h2;
    else
      prev.link = h2;
  }
  return head;
}
```

Question 5. [10 MARKS]

Consider the following class for nodes in a Binary Search Tree:

```
class BSTNode {
  public Comparable key;
  public BSTNode left;
  public BSTNode right;
}
```

Complete the following method. **Do not use recursion**. You can use helper methods, but they cannot be recursive. Do not use any other datatypes such as arrays or lists.

Recall that the `Comparable` interface consists of the method `int compareTo(Object o)`.

```
/** Return the second smallest key in the binary search tree rooted at t.
    Precondition: t contains at least two keys, and all keys are distinct. */
public static Comparable secondSmallest(BSTNode t) {

  BSTNode prev = null;
  BSTNode sm = t;
  // Follow left links to smallest.
  while (sm.left != null) {
    prev = sm;
    sm = sm.left;
  }
  if (sm.right != null)
    return smallest(sm.right);
  else
    return prev.key;
}

public static Comparable smallest(BSTNode s) {
  while (s.left != null) {
    s = s.left;
  }
  return s.key;
}
```

Question 6. [14 MARKS]

Consider the following class for nodes in a Binary Search Tree:

```
class BSTNode {
  public Comparable key;
  public BSTNode left;
  public BSTNode right;
}
```

Complete the following method. You can use a helper methods. You **must** use recursion. Do not use any other datatypes such as arrays or lists.

Recall that the `Comparable` interface consists of the method `int compareTo(Object o)`.

```
  /** Return true exactly when the tree rooted at t is a binary search tree.
   *  Precondition:  All the keys in the tree are distinct.  */
  public static boolean isBST(BSTNode t) {

    if (t == null)
      return true;
    return isBSTHelper(t, null, null);
  }

  /** Recursively check that the subtree root t has a key between the min
   *  (mn) and max (mx) values of its ancestors. */
  public boolean isBSTHelper(BSTNode t, Comparable mn, Comparable mx) {
    boolean check = true;
    if ((mn != null) && (t.key.compareTo(mn) < 0))
      check = false;
    if ((mx != null) && (t.key.compareTo(mx) > 0))
      check = false;
    if (check && t.left != null)
      check = isBSTHelper(t.left, mn, t.key);
    if (check && t.right != null)
      check = isBSTHelper(t.right, t.key, mx);
    return check;
  }
```

For scratch work.

Total Marks = 78