CSC238:Discrete Mathematics for Computer Science – Spring 2003

# Assignment 3: Propositional Logic.

*Due: 10 a.m., Thurs., March. 20*
*This assignment is worth 10 percent of the total marks for this course.*

This assignment provides basic exercises in propositional logic. We will mark only a (secret) subset of the questions below. As always in this course, your proofs will be marked for correctness, along with brevity and clarity.

Your answers can be handwritten. Use standard 8.5 by 11 inch paper. Please staple all the sheets together, and hand them to your tutor at the **beginning** of the tutorial on the due date. If you cannot make it to that tutorial, then leave your assignment at your instructor's office (Pratt, Room 283) before 10am on the due date. Since the tutors will be discussing the solutions in the tutorial immediately after your assigments are due, we will not accept late assignments (the course homepage describes what you should do in case of medical or other emergencies which prevent you from completing an assignment on time).

1. State whether or not each of the following assertions is true, and justify your answer (in particular, you can use a truth table):

   (a) $(x \to y) \land (x \to z)$ is logically equivalent to $x \to (y \land z)$.

   (b) $(y \to x) \land (z \to x)$ is logically equivalent to $(y \land z) \to x$.

2. Use only the logical equivalences in Section 5.6 of the lecture notes (and, in particular, **do not use truth tables**) to prove:

   (a) $(x \leftrightarrow \neg y) \to z$ is logically equivalent to $(x \land y) \lor (\neg x \land \neg y) \lor z$ .

   (b) $(x \leftrightarrow \neg y) \to \neg(x \to y)$ is logically equivalent to $y \to x$ .

   (c) $(x \land \neg y) \to \neg z$ is logically equivalent to $(x \land z) \to y$ .

3. Suppose $P$ and $Q$ are two unstatisfiable propositional formulas, $R$ is a tautology, and $S$ is satisfiable. For each of the statements below, decide whether or not it is true, and prove your answer.

   (a) $P \leftrightarrow Q$ is a tautology.

   (b) $P$ logically implies $R$.

   (c) $R$ logically implies $P$.

(d) $P \leftrightarrow R$ is unsatisfiable.

(e) $S \rightarrow P$ is satisfiable.

4. Write two propositional formulas, one in DNF the other in CNF, that are logically equivalent to $((x \rightarrow y) \vee (\neg x \wedge z)) \leftrightarrow (y \vee z)$. Justify your answers.

5. Prove that $\{\downarrow\}$ (i.e. nor, as described in the lecture notes) is a complete set of connectives.

6. Is $\{\bigoplus, \wedge, \vee\}$ a complete set of connectives? Here $\bigoplus$ denotes the "exclusive or" connective described in the lecture notes. Prove your answer.