

Introduction to spaghetti and meatballs



Course web site (includes course information sheet):

<http://www.dgp.toronto.edu/~karan/courses/418/fall12016>

Instructors:

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Textbooks: Fundamentals of Computer Graphics
OpenGL Programming Guide & Reference

Tutorials: (first tutorial next week)

Today's Topics

0. Introduction: What is Computer Graphics?
1. Basics of scan conversion (line drawing)
2. Representing 2D curves



Topic 0.

**Introduction:
What Is Computer Graphics?**

What is Computer Graphics?

Computers:

accept, process, transform and present information.

Computer Graphics:

accept, process, transform and present information
in a visual form.

Ok but... what is the course really about?

The science of turning the rules of geometry, motion and physics into (digital) pictures that mean something to people

What its not about?

Photoshop, AutoCAD, Maya, Renderman, Graphics APIs.

...**wow**, heavy math and computer science!!

Movies

Movies define directions in CG
Set quality standards
Driving medium for CG



Games

Games emphasize the interactivity and AI
Push CG hardware to the limits (for real time performance)



Design

CG for prototyping and fabrication
Requires precision modeling and engineering visualization



Scientific and Medical Visualization, Operation

Requires handling large datasets
May need device integration
Real-time interactive modeling & visualization



GUIs, AR/VR, scanners...

Interaction with software & hardware, I/O of 3D data
Emphasis on usability



Computer Graphics: Basic Questions

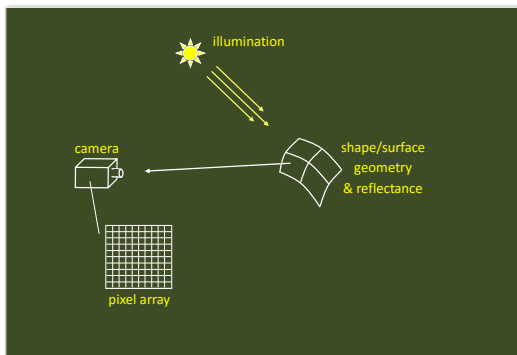
- Form (modeling)
 - How do we represent (2D or 3D) objects & environments?
 - How do we build these representations?
- Function, Behavior (animation)
 - How do we represent the way objects move?
 - How do we define & control their motion?
- Appearance (rendering)
 - How do we represent the appearance of objects?
 - How do we simulate the image-forming process?

What is an Image?

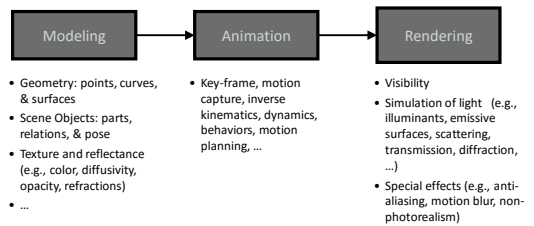
Image = distribution of light energy on 2D "film"
 Digital images represented as rectangular arrays of pixels



Form & Appearance in CG

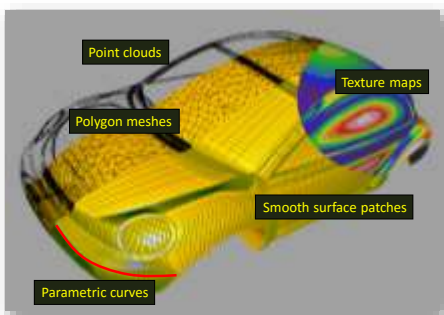


The Graphics Pipeline

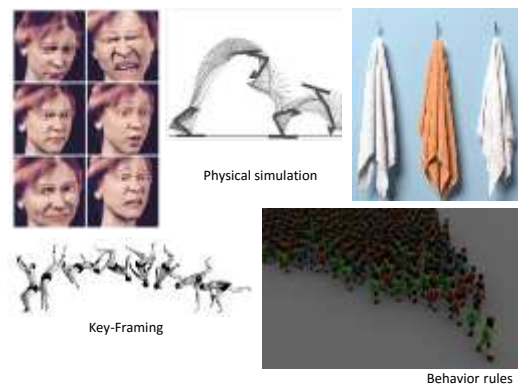


Graphics Pipeline: Modeling

How do we represent an object geometrically on a computer?



Graphics Pipeline: Animation



Graphics Pipeline: Rendering



Input: Scene description, lighting, camera

Output: Image that the camera will observe...
accounting for visibility, clipping, projection,...

What You Will Take Away ...

#1: yes, math IS useful in CS !!

#2: how to turn math & physics into pictures.

#3: basics of image synthesis

#4: how to code CG tools

Topic 1.

Basic Raster Operations: Line Drawing

- A simple (but inefficient) line drawing algorithm
- Bresenham's algorithm
- Line anti-aliasing

Course Topics

Principles

Theoretical & practical foundations of CG
(core mathematics, physics, modeling methods)

CG programming (assignments & tutorials)

- Experience with OpenGL (industry-standard CG library)
- Creating CG scenes

Administrivia

Grading:

- 50%: 3 assignments handed out in class (25% 15% 10%).
- 50%: 1 test in class (15%) + 1 final exam (35%).
- First assignment: on web in two weeks.
- Wooden Monkey assignment on web now!
- Check web for schedule, dates, more details & policy on late assignments.

Tutorial sessions:

- Math refreshers, OpenGL tutorials, additional topics.
- Attendance **STRONGLY** encouraged since I will not be lecturing on these topics in class.

Lecture slides & course notes, already on web.

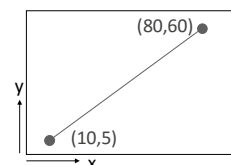
2D Drawing

Common geometric primitives:



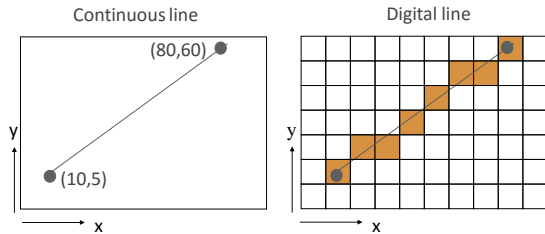
When drawing a picture, 2D geometric primitives are specified as if they are drawn on a continuous plane

Drawing command:
Draw a line from point (10,5)
to point (80,60)



2D Drawing

In reality, computer displays are arrays of pixels, not abstract mathematical continuous planes



In graphics, the conversion from continuous to discrete 2D primitives is called scan conversion or rasterization

Basic Raster Operations (for 2D lines)

- **Scan conversion:** Given a pair of pixels defining the line's endpoints & a color, paint all pixels that lie on the line.
- **Clipping:** If one or more endpoints is out of bounds, paint only the line segment that is within bounds.
- **Region filling:** Fill in all pixels within a given closed connected boundary of pixels.

Line Scan Conversion: Key Objectives

Accuracy:

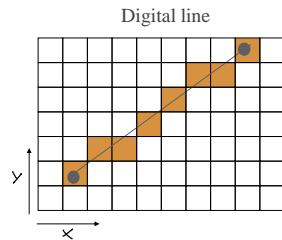
pixels should approximate line closely.

Speed:

line drawing should be efficient

Visual Quality:

No discernable "artifacts".



Equation of a Line

Explicit : $y = mx + b$

Parametric :

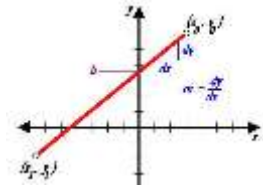
$$x(t) = x_0 + (x_1 - x_0) * t$$

$$y(t) = y_0 + (y_1 - y_0) * t$$

$$P = P_0 + (P_1 - P_0) * t$$

$$P = P_0 * (1-t) + P_1 * t \text{ (weighted sum)}$$

Implicit : $(x-x_0)dy - (y-y_0)dx = 0$



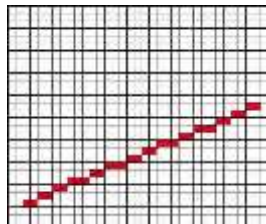
Algorithm I

DDA (Digital Differential Analyzer)

Explicit form:

$$y = dy/dx * (x-x_0) + y_0$$

```
float y;
int x;
dx = x1-x0; dy = y1 - y0;
m = dy/dx;
y = y0;
for ( x=x0; x<=x1; x++)
{
    setpixel (x, round(y));
    y = y + m;
}
```



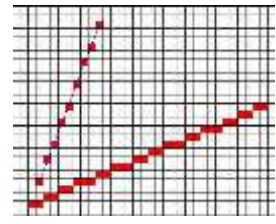
Algorithm I (gaps when m>1)

DDA (Digital Differential Analyzer)

Explicit form:

$$y = dy/dx * (x-x_0) + y_0$$

```
float y;
int x;
dx = x1-x0; dy = y1 - y0;
m = dy/dx;
y = y0;
for ( x=x0; x<=x1; x++)
{
    setpixel (x, round(y));
    y = y + m;
}
```



Algorithm II

Bresenham Algorithm

Slope is rational (ratio of two integers), $m = (y_1 - y_0) / (x_1 - x_0)$.

Assume line slope < 1 (first quadrant), implying that either

$$y_{i+1} = y_i \text{ or } y_{i+1} = y_i + 1.$$

We want to make this decision using only integer math.

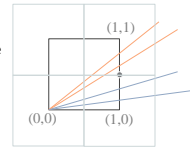
Algorithm II

Bresenham Algorithm: *Implicit View*

$$f(x,y) = x \cdot dy - y \cdot dx = 0 \quad // \text{ for points on the line}$$

$$> 0 \quad // \text{ below the line}$$

$$< 0 \quad // \text{ above the line}$$



$$f(x+1, y+0.5) = f(x,y) + dy - 0.5 \cdot dx$$

$$\text{-ve} \rightarrow, \text{ +ve} \nearrow$$

$$f(1, 0.5) = dy - 0.5 \cdot dx$$

$$\text{-ve: pick } (1,0)$$

$$\text{+ve: pick } (1,1)$$

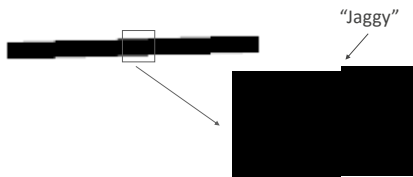
$$\text{err} = 2f(x+1, y+0.5) = 2f(x,y) + 2dy - dx \quad // \text{ getting rid of the float}$$

$$\text{-ve: } 2f(x+1, y) = 2f(x,y) + 2dy \quad \text{err}' = \text{err} + 2dy$$

$$\text{+ve: } 2f(x+1, y+1) = f(x,y) + dy - dx \quad \text{err}' = \text{err} + 2dy - 2dx$$

Aliasing

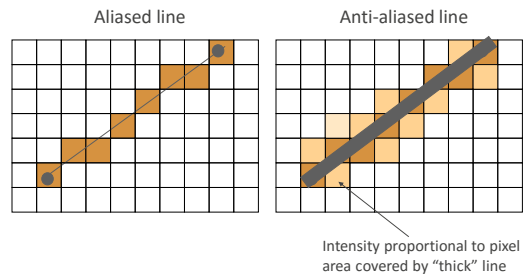
Raster line drawing can produce a "jaggy" appearance.



- Jaggies are an instance of a phenomenon called aliasing.
- Removal of these artifacts is called anti-aliasing.

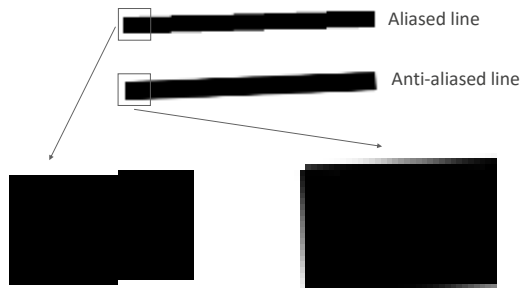
Anti-Aliasing

How can we make a digital line appear less jaggy?



Main idea: Rather than just drawing in 0's and 1's, use "in-between" values in neighborhood of the mathematical line.

Anti-Aliasing: Example



Topic 2.

2D Curve Representations

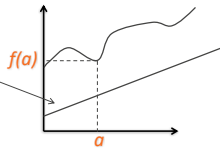
- Explicit representation
- Parametric representation
- Implicit representation
- Tangent & normal vectors

Explicit Curve Representations: Definition

Curve represented by a function f
such that:

$$y=f(x)$$

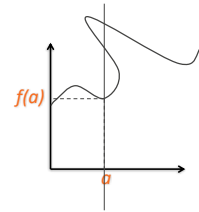
$$\text{line: } y=mx+b$$



Explicit Curve Representations: Limitations

Curve represented by a function f
such that:

$$y=f(x)$$



Parametric Curve Representation: Definition

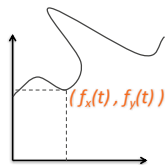
Curve represented by two functions f_x, f_y

And an interval $[a,b]$

such that:

$$(x,y)=(f_x(t), f_y(t))$$

are points on the curve for
 t in $[a,b]$

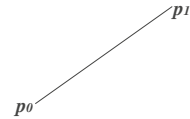


Parametric Representation of a Line Segment

$$p(t) = p_0 + (p_1 - p_0)t, \quad 0 \leq t \leq 1$$

$$0 \leq t \leq \infty : \text{ray from } p_0 \text{ through } p_1$$

$$-\infty \leq t \leq \infty : \text{line through } p_0 \text{ and } p_1$$

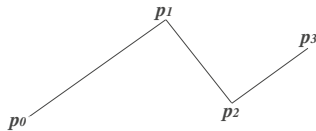


A curve is closed when ??

In general if $p(t) = a_0 + a_1t$, how do you solve for a_0, a_1 ?

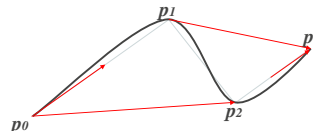
Line Segment as interpolation

$$p(t) = a_0 + a_1t$$



Curve as interpolation (Catmull-Rom)

$$p(t) = a_0 + a_1t + a_2t^2 + a_3t^3$$



Polygons

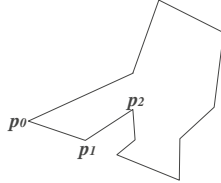
Polygon: A continuous piecewise linear closed curve.

Simple polygon: non-self intersecting.

Convex: all angle less than 180 degrees.

Regular: simple, equilateral, equiangular.

$$n\text{-gon: } p_i = r(\cos(2\pi i/n), \sin(2\pi i/n)), \quad 0 \leq i < n$$



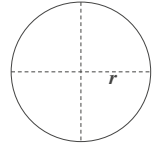
Representations of a Circle

Parametric:

$$p(t) = r(\cos(2\pi t), \sin(2\pi t)), \quad 0 \leq t \leq 1$$

Implicit:

$$x^2 + y^2 - r^2 = 0$$



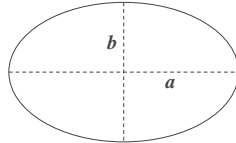
Representations of an Ellipse

Parametric:

$$p(t) = (a \cdot \cos(2\pi t), b \cdot \sin(2\pi t)), \quad 0 \leq t \leq 1$$

Implicit:

$$x^2/a^2 + y^2/b^2 - 1 = 0$$



Curve tangent and normal

Parametric:

$$p(t) = (x(t), y(t)). \quad \text{Tangent: } (x'(t), y'(t)).$$

Implicit:

$$f(x, y) = 0. \quad \text{Normal: } \text{gradient}(f(x, y)).$$

Tangent and normal are orthogonal.