CSC384 Test 2 (Backtracking Search, Knowledge Representation) Sample Questions

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1. Consider the following CSP with 3 variables X, Y and Z:

   $Dom(X) = \{1, \cdots, 10\}$, $Dom(Y) = \{5, \cdots, 15\}$, and $Dom(Z) = \{5, \cdots, 20\}$,

   and binary constraints: $C(X, Y) : X > Y$, $C(Y, Z) : Y + Z = 12$, and $C(X, Z) : X + Z = 16$.

   a) Draw the constraint graph.

   b) Are the constraints *arc consistent*? If no, apply arc consistency method repeatedly so they become arc consistent. What is the updated domain of each variable?

2. Consider the N-Queens problem. That is, the problem of placing N Queens on an $N \times N$ chessboard so that no two Queens can attack each other.

   Find the *first solution* to the 5-Queens problem by using the *Forward Checking* algorithm with *dynamic variable reordering* using the heuristic that we always instantiate next that variable with smallest remaining number of elements in its domain, breaking ties in favor of the lowest numbered variable.

   (Note, use the same CSP formulation as that used in class. That is, we have 5 variables, $Q_1, \ldots, Q_5$ each with domain $[1, 2, 3, 4, 5]$. Each variable Q_i represents the queen in the i-th row, and the assignment $Q_i = j$ means that the queen in the i-th row has been placed in the j-th column.

   Draw the search tree explored by this algorithm. **At each node** indicate

   (a) The variable being instantiated, and the value it is being assigned.

   (b) A list of the variables that have had at least one of their values pruned by the new assignment, and for such variable a list of its remaining legal values. *Note, you must follow the forward checking algorithm precisely: only prune values that would be pruned by the algorithm.*

   (c) Mark any node where a deadend occurs because of a domain wipe out (use the symbol DWO).

3. Translate the following English sentences into logic:

   (a) All horses, cows, and pigs are mammals.

   (b) Offspring and Parent are inverse relations.

   (c) Not all basketball players are tall.

4. Consider a first-order language $\mathcal{L}$ containing the following basic symbols:

   - Constants, A, B, C, D.

- The binary predicate R.
- The unary predicates P and Q.

Let $\mathcal{M}$ be a model for $\mathcal{L}$, with domain $D = \{a, b, c, d\}$, and interpretation functions Φ and Ψ:

(a) $\Phi(A) = a$, $\Phi(B) = b$, $\Phi(C) = c$, $\Phi(D) = d$.
(b) $\Psi(R) = \{(b, a), (c, d)\}$.
(c) $\Psi(P) = \{b, c\}$.
(d) $\Psi(Q) = \{a, d\}$.

Which of the following formulas are satisfied by $\mathcal{M}$.

(a) $R(C, B) \vee R(B, A)$
(b) $\forall x.P(x) \wedge \neg Q(x)$.
(c) $\forall x.P(x) \rightarrow \neg\exists y.R(y, x)$.
(d) $\forall x.Q(x) \rightarrow \neg\exists y.R(y, x)$.
(e) $\forall x.Q(x) \rightarrow \exists y.R(y, x)$.
(f) $\exists x, y.(P(x) \wedge Q(y)) \rightarrow R(y, x)$.

5. Give the most general unifier (MGU) for the following pairs of expressions, or say *why* it does not exist. In all of these expressions variables start with a "?".

(a) $P(A, B, B)$, $P(?x, ?y, ?z)$
(b) $Older(Father(?y), ?y)$, $Older(Father(?x), John)$
(c) $Knows(Father(?y), ?y)$, $Knows(?x, ?x)$

6. Consider the following sentences:

- John likes all kinds of food.
- Apples are food.
- Chichen is food.
- Anything anyone eats and isn't killed by is food.
- Bill eats peanuts and is still alive.
- Sue eats everything Bill eats.

(a) Represent these sentences in first order logic.
(b) Convert the formulas to clausal form. Indicate any Skolem functions or constants used.
(c) Convert the negation of the statement "What food does Sue eat" to clause form (using an answer literal).
(d) Answer this question (i.e., "What food does Sue eat") using reoslution and answer extraction. In the proof use *the notation developed in class!*. That is, every new clause must be labeled by the resolution step that was used to generate it. For example, a clause labled $R[4c, 1d]\{x = a, y = f(b)\}$ means that it was generated by resolving literal c of clause 4 against literal d of clause 1, using the MGU $\{x = a, y = f(b)\}$.