Missing Data Problems and Collaborative Filtering

CSC2535

Richard Zemel & Ben Marlin

Department of Computer Science
University of Toronto

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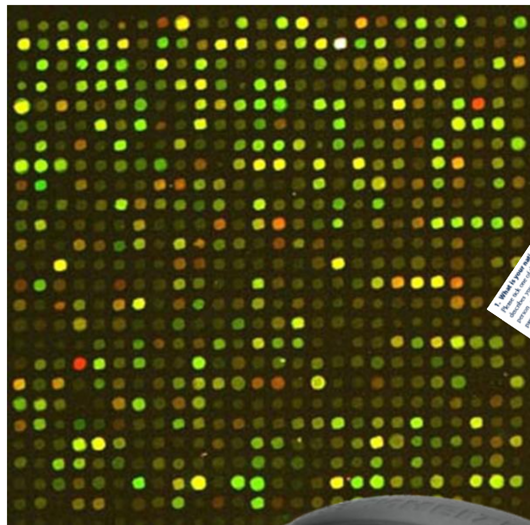
Contents:

Introduction



Introduction

Collaborative filtering – users assign ratings to items → system uses information from all users to recommend previously unseen items that a user might like

One approach to recommendation: predict ratings for all unrated items, recommend highest predicted ratings

Critical assumption: missing ratings are missing at random

One way to violate: value of variable affects probability value will be missing – bias in observed ratings, and hence learned parameters

Also complementary bias in standard testing procedure – distribution of observed data different from distribution of complete data, so estimated error on observed test data poor estimate of complete data error

Introduction: Survey Sampling Example

Data Variables	X	Answers to questions.
Response Variables	R	Did the respondent answer the question?

Introduction: Medical Diagnosis Example

Latent Variables — Z — Diseases

Data Variables — X — Symptoms

Response Variables — R — Was a test performed to check for the symptom?

Introduction: Recommender Systems Example

Latent Variables	Z	Preferences and Tastes
Data Variables	X	Ratings or Purchase History
Response Variables	R	Did the user rate or buy the item?

Introduction: Basic Notation

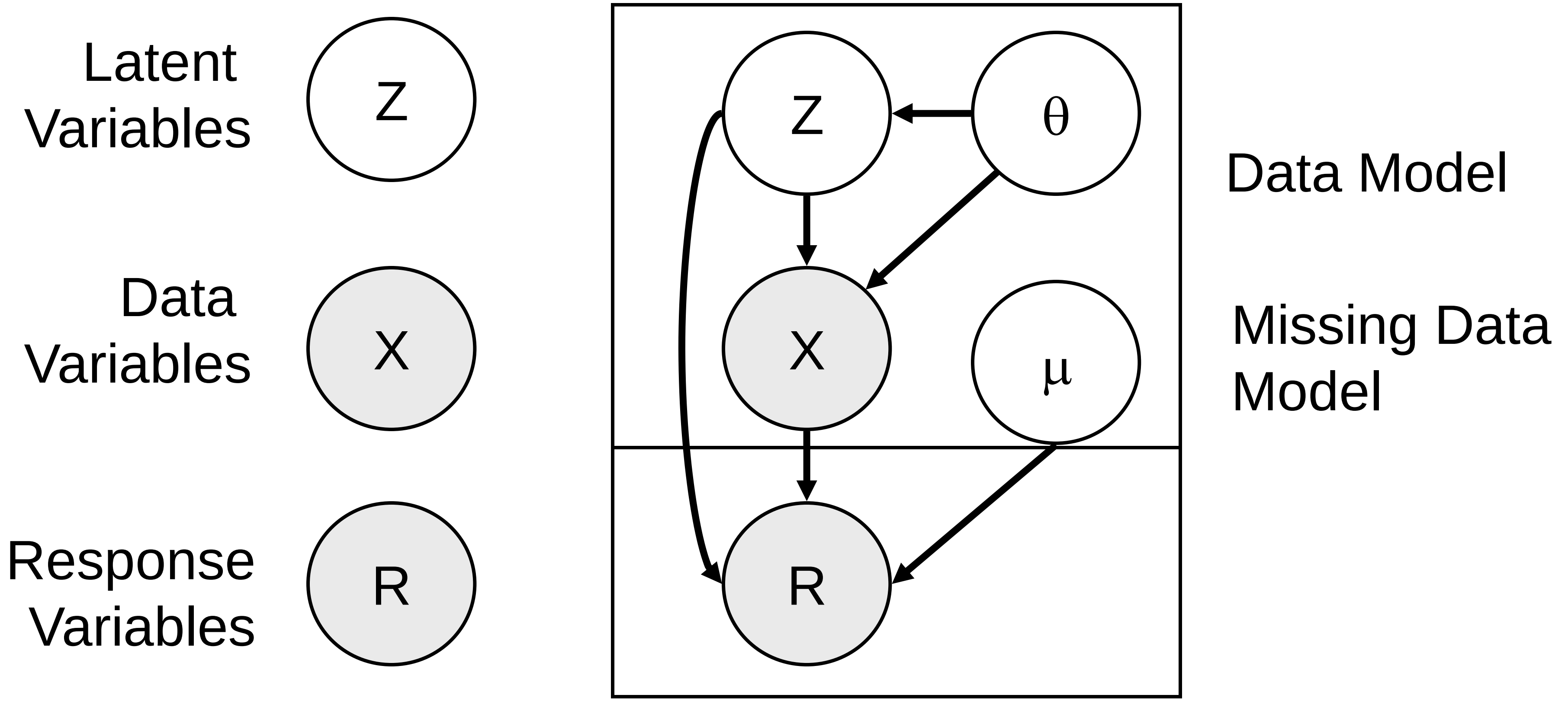
N	Number of data cases.
D	Number of data dimensions.
C	Number of classes.
V	Number of multinomial values.
K	Number of clusters or hidden units.

Introduction: Notation for Missing Data

$\mathbf{x}_n$	0.1 | 0.9 | 0.2 | 0.7 | 0.3	Data Vector
$\mathbf{r}_n$	1 | 0 | 0 | 1 | 1	Response Vector
$\mathbf{o}_n$	1 | 4 | 5	Observed Dimensions
$\mathbf{m}_n$	2 | 3	Missing Dimensions
$\mathbf{x}_n^{\mathbf{o}_n}, \mathbf{x}_n^o$	0.1 | 0.7 | 0.3	Observed Data
$\mathbf{x}_n^{\mathbf{m}_n}, \mathbf{x}_n^m$	0.9 | 0.2	Missing Data

Contents:

Theory of Missing Data: Generative Process



Theory of Missing Data: Factorizations

Data/Selection Model Factorization:

$$P(\mathbf{x}, \mathbf{r}, \mathbf{z}|\theta, \mu) = P(\mathbf{r}|\mathbf{x}, \mathbf{z}, \mu)P(\mathbf{x}, \mathbf{z}|\theta)$$

• The probability of selection depends on the true values of the data variables and latent variables.

Pattern Mixture Model Factorization:

$$P(\mathbf{x}, \mathbf{r}, \mathbf{z}|\vartheta, \nu) = P(\mathbf{x}, \mathbf{z}|\mathbf{r}, \vartheta)P(\mathbf{r}|\nu)$$

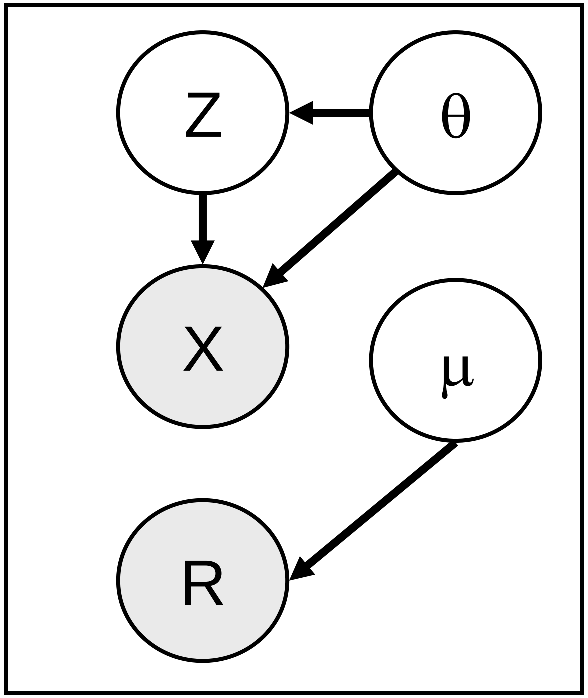
• Each response vector defines a different pattern, and each pattern has a different distribution over the data.

Theory of Missing Data: Classification

Missing Completely at Random:

• Response probability is independent of data variables and latent variables.

$$P(\mathbf{r}|\mathbf{x}, \mathbf{z}, \mu) = P(\mathbf{r}|\mu)$$



MCAR Examples:

Send questionnaires to a random subset of the population or use random digit dialing.

Theory of Missing Data: Classification

Missing at Random:

• Typically written in a short-hand form that looks like a statement of probabilistic independence:

$$P(\mathbf{r}|\mathbf{x}, \mathbf{z}, \mu) = P(\mathbf{r}|\mathbf{x}^o, \mu)$$

• MAR is actually a different type of condition that requires a particular set of symmetries hold in P(r|x,z,μ):

$$P(\mathbf{r}|\mathbf{x}^{o(\mathbf{r})}, \mathbf{x}^{m(\mathbf{r})}, \mathbf{Z}, \mu) = f(\mathbf{r}, \mathbf{x}^{o(\mathbf{r})}, \mu) \text{ ... for all } \mathbf{x}^m$$

Theory of Missing Data: Classification

Missing at Random Examples:

Respondents are not required to provide information about their employer if they are not currently employed.

Doctor only orders test B if the result of test A was negative. If result of test A is positive, result for test B is missing.

Theory of Missing Data: Classification

What Does it mean to be Missing at Random?

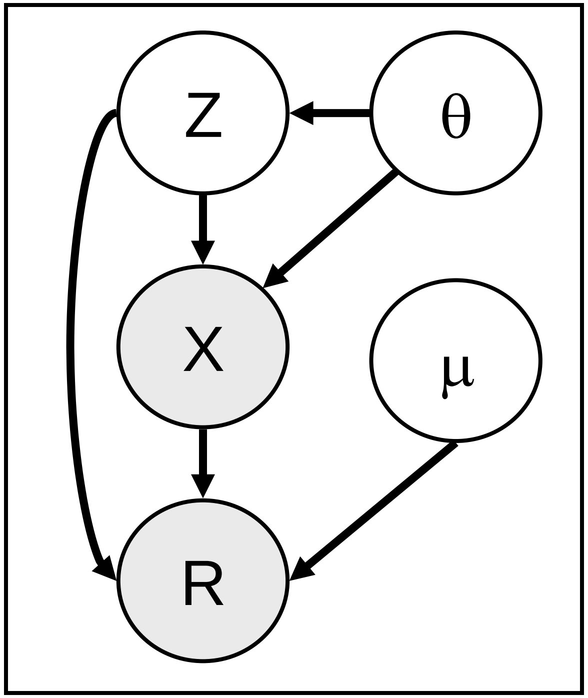
- MAR is ***not*** a statement of independence between random variables. MAR requires that particular symmetries hold so that P(R=r|X=x) can be determined from observed data only.

$X \backslash R$	$0\ 0$	$0\ 1$	$1\ 0$	$1\ 1$
$0\ 0$	α	β	γ	$1-\alpha-\beta-\gamma$
$0\ 1$	α	δ	γ	$1-\alpha-\delta-\gamma$
$1\ 0$	α	β	λ	$1-\alpha-\beta-\lambda$
$1\ 1$	α	δ	λ	$1-\alpha-\delta-\lambda$

Theory of Missing Data: Classification

Not Missing at Random:

• Allows for arbitrary dependence of response probabilities on missing data values and latent variables:



$$P(\mathbf{r}|\mathbf{x}, \mathbf{z}, \mu)$$ **No Simplifications**

An Easy Way to Violate MAR:

• Let the probability that a data variable is observed depend on the value of that data variable.

Theory of Missing Data: Classification

Not Missing at Random Examples:

Snowfall reading is likely to be missing if weather station is covered with snow.

Participants in a longitudinal health study for a heart medication may die of a heart attack during the study.

Users are more likely to rate or buy items they like than items they don’t like.

Theory of Missing Data: Inference

MCAR/MAR Posterior:

$$P(\theta|\mathbf{x}^o, \mathbf{r}) \propto \int\int\int P(\mathbf{x}, \mathbf{z}|\theta)P(\mathbf{r}|\mathbf{x}, \mathbf{z}, \mu)P(\theta|\omega)P(\mu|\eta)d\mu dZ d\mathbf{x}^m$$

$$\propto \int f(\mathbf{r}, \mathbf{x}, \mu)P(\mu|\eta)d\mu \cdot \int\int P(\mathbf{x}, \mathbf{z}|\theta)P(\theta|\omega)dZ d\mathbf{x}^m$$

$$\propto P(\mathbf{x}^o|\theta)P(\theta|\omega)$$

- When MCAR or MAR holds, the posterior can be greatly simplified. Inference for θ does not depend on r, μ, or η. The missing data can be *ignored*.

Theory of Missing Data: Inference

NMAR Posterior:

$$P(\theta|\mathbf{x}^o, \mathbf{r}) \propto \int\int\int P(\mathbf{x}, \mathbf{z}|\theta)P(\mathbf{r}|\mathbf{x}, \mathbf{z}, \mu)P(\theta|\omega)P(\mu|\eta)d\mu dZ d\mathbf{x}^m$$

- When MAR fails to hold, the posterior does not simplify.

- Basing inference on the observed data posterior and ignoring the missing data model leads to provably biased inference for data model parameters.

Contents:

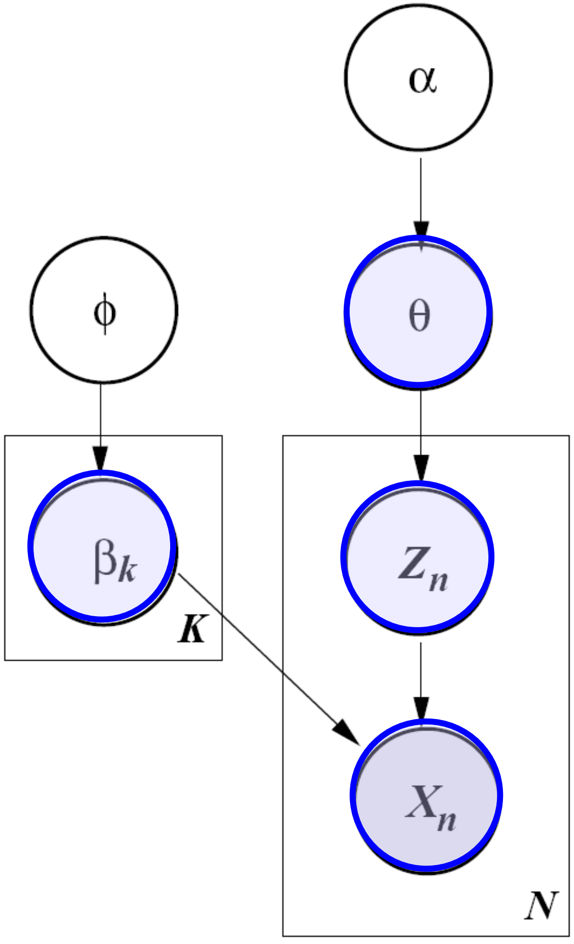
Multinomial Models: Mixture

Probability Model:

$$P(Z_n = k|\theta) = \theta_k$$

$$P(\mathbf{X}_n = \mathbf{x}_n|Z_n = k, \beta) = P(\mathbf{x}_n|\beta_k)$$

$$P(\theta, \beta|\alpha, \phi) = P(\theta|\alpha)\prod_k P(\beta_k|\phi)$$



Properties:

• Allows for a fixed, finite number of clusters.

• In the multinomial mixture, $P(x_n|\beta_k)$ is a product of discrete distributions. The prior on β and θ is Dirichlet.

Multinomial Models: Mixture

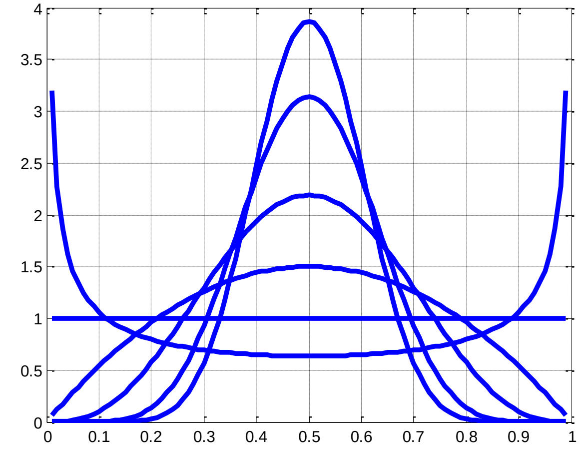
Dirichlet Distribution:

Bayesian mixture modeling becomes much easier when conjugate priors are used for the model parameters. The conjugate prior for the mixture proportions θ is the Dirichlet distribution.

$$P(\theta|\alpha) = \frac{\Gamma(\sum_k \alpha_k)}{\prod_k \Gamma(\alpha_k)} \prod_k \theta_k^{\alpha_k - 1}$$

$$E[\theta_k|\alpha] = \frac{\alpha_k}{\sum_{k=1}^{K} \alpha_k}$$

$$P(\theta|\alpha, \mathbf{z}) = \frac{\Gamma(N + \sum_k \alpha_k)}{\prod_k \Gamma(C_k + \alpha_k)} \prod_k \theta_k^{C_k + \alpha_k - 1}$$



Multinomial Models: Mixture

MAP EM Algorithm:

$$\text{E-Step: } q_n(k) \leftarrow \frac{\theta_k \prod_{d=1}^{D} \prod_{v=1}^{V} \beta_{vdk}^{[r_{dn}=1][x_{dn}=v]}}{\sum_{k'=1}^{K} \theta_{k'} \prod_{d=1}^{D} \prod_{v=1}^{V} \beta_{vdk'}^{[r_{dn}=1][x_{dn}=v]}}$$

$$\text{M-Step: } \theta_k \leftarrow \frac{\alpha_k - 1 + \sum_{n=1}^{N} q_n(k)}{N - K + \sum_{k=1}^{K} \alpha_k}$$

$$\beta_{vdk} \leftarrow \frac{\phi_{vdk} - 1 + \sum_{n=1}^{N} q_n(k)[r_{dn} = 1][x_{dn} = v]}{\sum_{n=1}^{N} q_n(k)[r_{dn} = 1] - V + \sum_{v=1}^{V} \phi_{vdk}}$$

Multinomial Models: Mixture/CPT-v

Probability Model:

$$P(\theta|\alpha) = \mathcal{D}(\theta|\alpha)$$

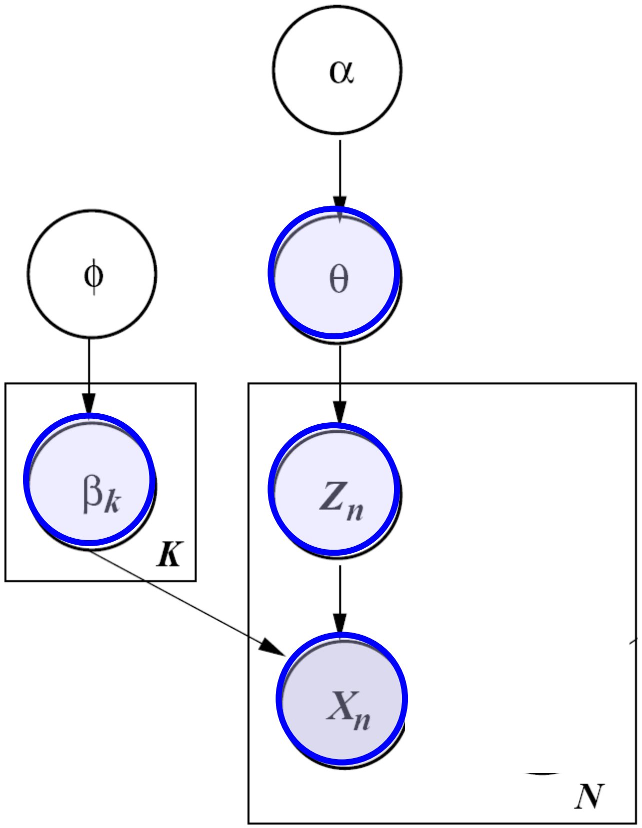
$$P(\beta|\phi) = \prod_{k=1}^{K}\prod_{d=1}^{D}\mathcal{D}(\beta_{dk}|\phi_{dk})$$

$$P(Z_n = k|\theta) = \theta_k$$

$$P(\mathbf{X} = \mathbf{x}_n|Z_n = k, \beta) = \prod_{d=1}^{D}\prod_{v=1}^{V}\beta_{vdk}^{[x_{dn}=v]}$$

$$P(\mu|\xi) = \prod_{v}\mathcal{B}(\mu_v|\xi_v)$$

$$P(\mathbf{R} = \mathbf{r}_n|\mathbf{X} = \mathbf{x}_n, \mu) = \prod_{d=1}^{D}\prod_{v=1}^{V}\mu_v^{[r_{dn}=1][x_{dn}=v]}(1-\mu_v)^{[r_{dn}=0][x_{dn}=v]}$$



Multinomial Models: Mixture/CPT-v

MAP EM Algorithm (E-Step):

$$q_n(k) = P(z_n = k|\mathbf{x}_n^o, \mathbf{r}_n, \theta, \beta, \mu)$$

$$= \frac{\theta_k \prod_{d=1}^{D} \left(\prod_{v=1}^{V} (\beta_{vdk}\mu_v)^{[x_{dn}=v]}\right)^{[r_{dn}=1]} \left(\sum_{v=1}^{V} \beta_{vdk}(1-\mu_v)\right)^{[r_{dn}=0]}}{\sum_{k=1}^{K} \theta_k \prod_{d=1}^{D} \left(\prod_{v=1}^{V} (\beta_{vdk}\mu_v)^{[x_{dn}=v]}\right)^{[r_{dn}=1]} \left(\sum_{v=1}^{V} \beta_{vdk}(1-\mu_v)\right)^{[r_{dn}=0]}}$$

$$q_n(k,v,d) = P(z_n = k, x_{dn} = v|\mathbf{x}_n^o, \mathbf{r}_n, \theta, \beta, \mu)$$

$$= q_n(k) \left(\frac{\mu_v \beta_{vdk}}{\sum_{v'=1}^{V} \mu_{v'}\beta_{v'dk}}\right)^{[r_{dn}=1]} \left(\frac{(1-\mu_v)\beta_{vdk}}{\sum_{v'=1}^{V}(1-\mu_{v'})\beta_{v'dk}}\right)^{[r_{dn}=0]}$$

Multinomial Models: Mixture/CPT-v

MAP EM Algorithm (M-Step):

$$\theta_k = \frac{\alpha_k - 1 + \sum_{n=1}^{N} q_n(k)}{N - K + \sum_{k=1}^{K} \alpha_k}$$

$$\beta_{vdk} = \frac{\phi_{vdk} - 1 + \sum_{n=1}^{N} q_n(k)[r_{dn} = 1][x_{dn} = v] + q_n(k, v, d)[r_{dn} = 0]}{\sum_{n=1}^{N} q_n(k) - V + \sum_{v=1}^{V} \phi_{vdk}}$$

$$\mu_v = \frac{\xi_{1v} - 1 + \sum_{n=1}^{N} \sum_{d=1}^{D} [r_{dn} = 1][x_{dn} = v]}{\xi_{1v} + \xi_{0v} - 2 + \sum_{n=1}^{N} \sum_{d=1}^{D} [r_{dn} = 1][x_{dn} = v] + q_n(v, d)[r_{dn} = 0]}$$

Other Models for Missing Data:

- K-Nearest Neighbors
- Probabilistic Principal Components Analysis
- Factor Analysis
- Mixtures of Gaussians
- Mixtures of PPCA/FA
- Probabilistic Matrix Factorization
- Maximum Margin Matrix Factorization
- Conditional Restricted Boltzmann Machines

Contents:

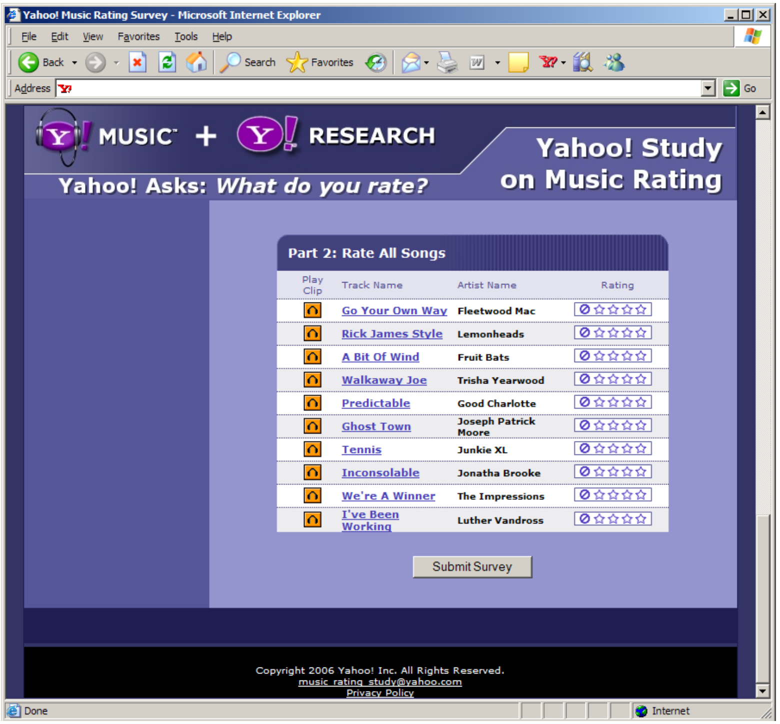
Collaborative Filtering:
Collaborative Prediction Problem

	★★☆	?	?	★★☆	★★☆
	?	★★☆	★★★	?	★★★
	★★★	?	★★☆	★★★	?

Collaborative Filtering : Yahoo!

Data was collected through an online survey of Yahoo! Music LaunchCast radio users.

- 1000 songs selected at random.
- Users rate 10 songs selected at random from 1000 songs.
- Answer 16 questions.
- Collected data from 35,000+ users.



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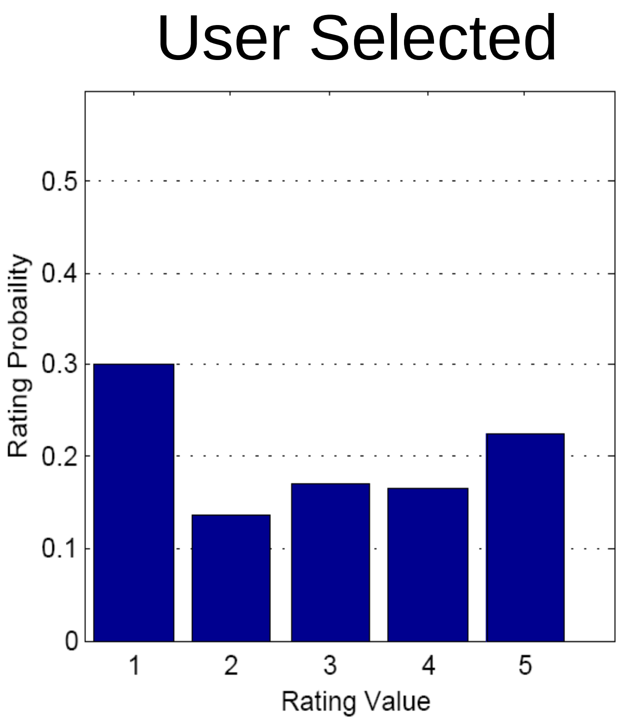
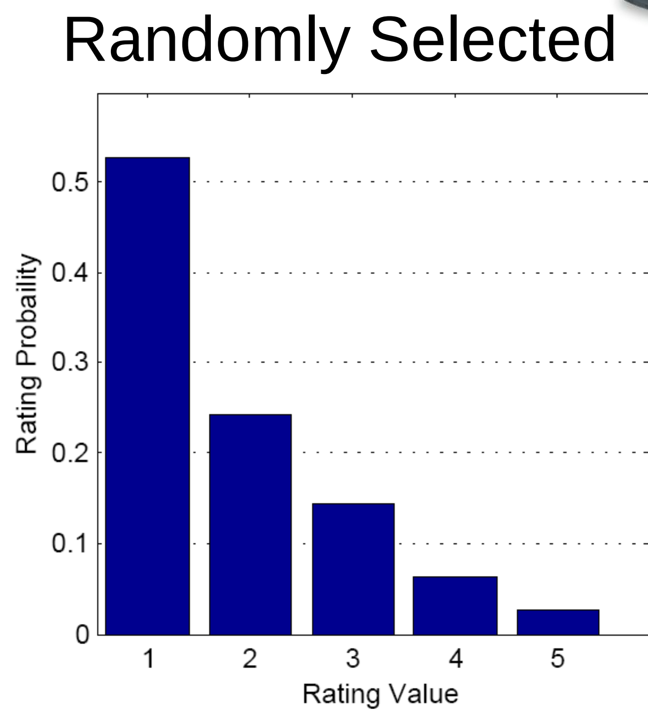
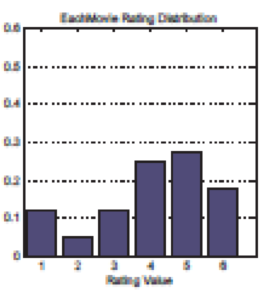
Collaborative Filtering: Yahoo!



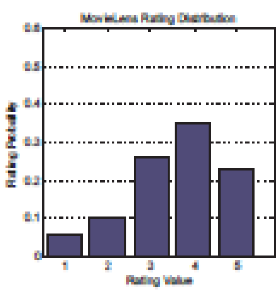


$$P_E^U(x=v|r=1) = \frac{\sum_i \sum_m [X_{im} = v][R_{im} = 1]}{\sum_i \sum_m [R_{im} = 1]} \qquad P_E^R(x=v|r=1)$$

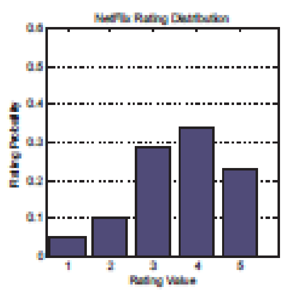
More Empirical Distributions



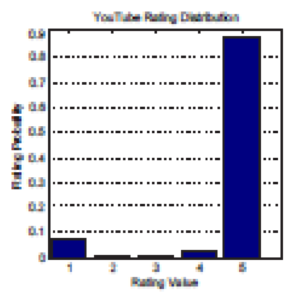
(a) EachMovie



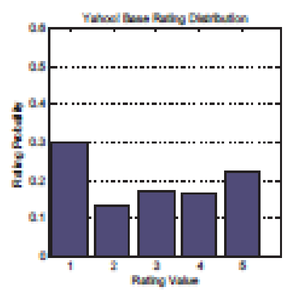
(b) MovieLens



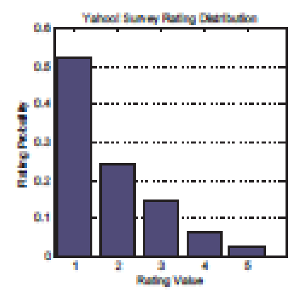
(c) NetFlix



(d) YouTube



(e) Yahoo! User

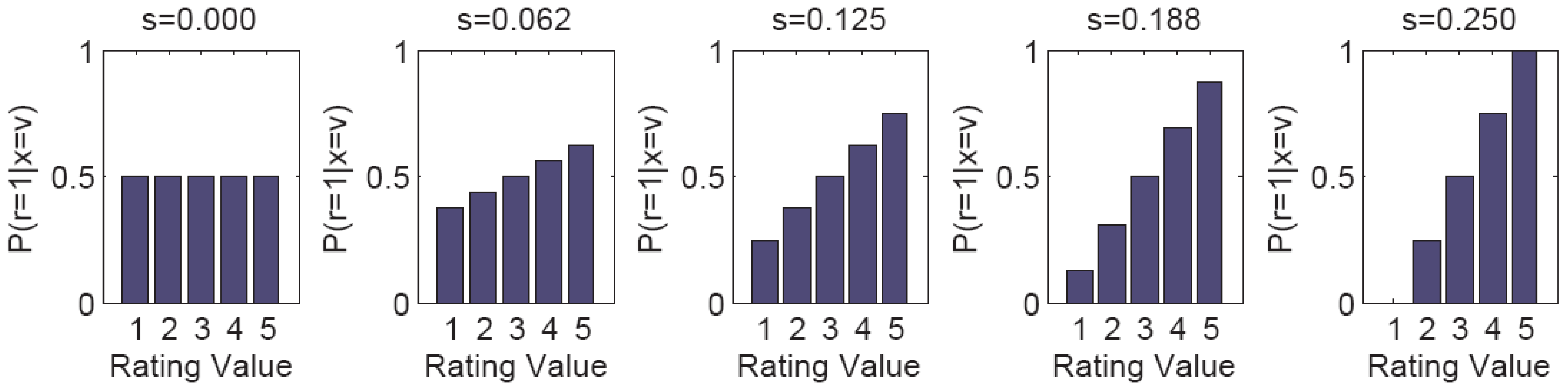


(f) Yahoo! Random

Collaborative Filtering: Jester

Jester gauge set of 10 jokes used as complete data. Synthetic missing data was added.

- 15,000 users randomly selected
- Missing data model: $\mu_v(s) = s(v-3)+0.5$



Experimental Protocol

Randomly partition users into 5 blocks of 1080 users

Three sets of ratings:

1. Observed ratings – all but one of original ratings
2. Test ratings for user-selected – remaining one
3. Test ratings for randomly-selected – ten survey responses

User-selected items – same distribution as observed

Randomly selected test items -- MCAR

Experimental Protocol

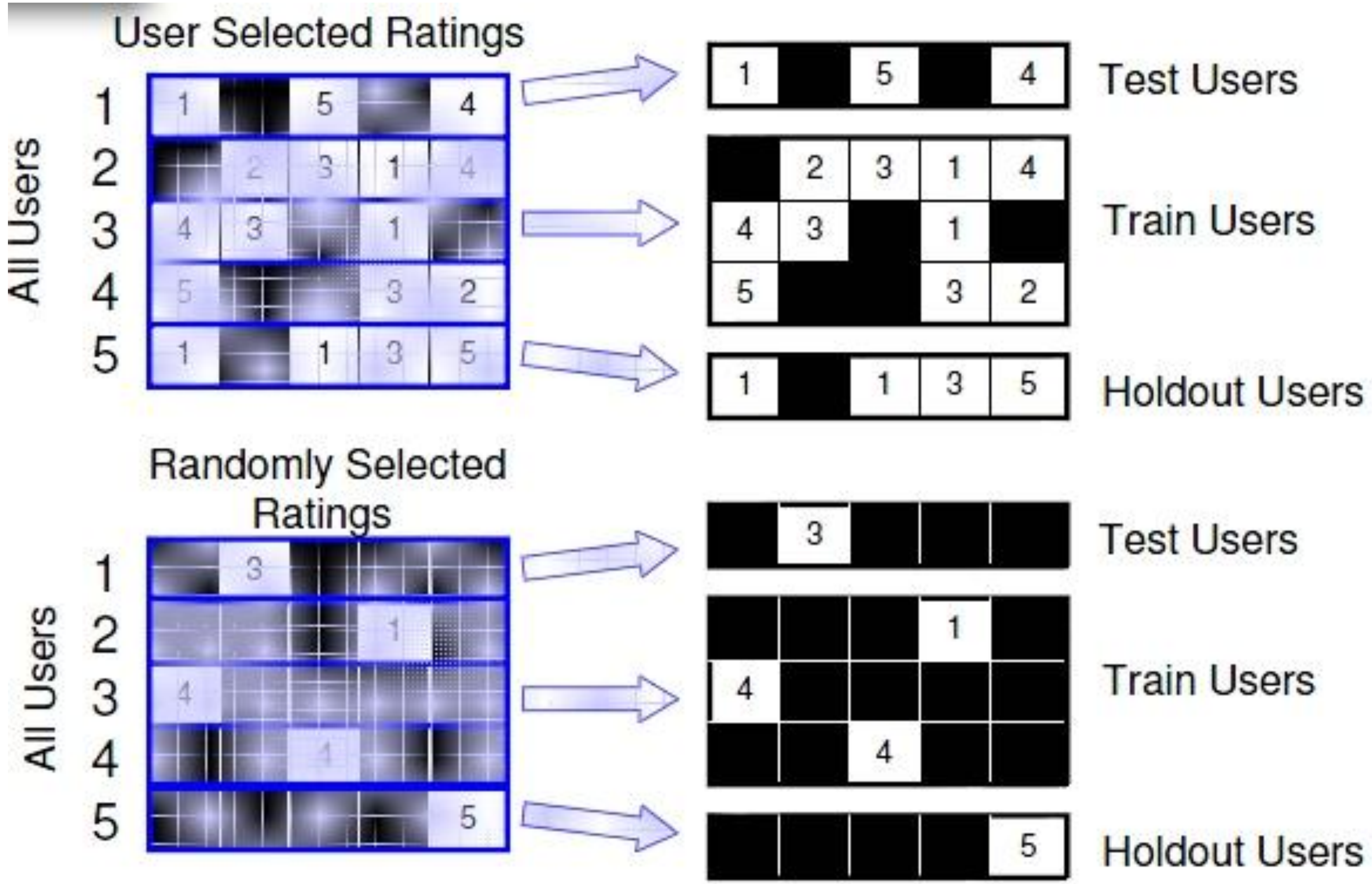
Weak Generalization

- Learn on training user observed ratings
- Evaluate on training user test ratings

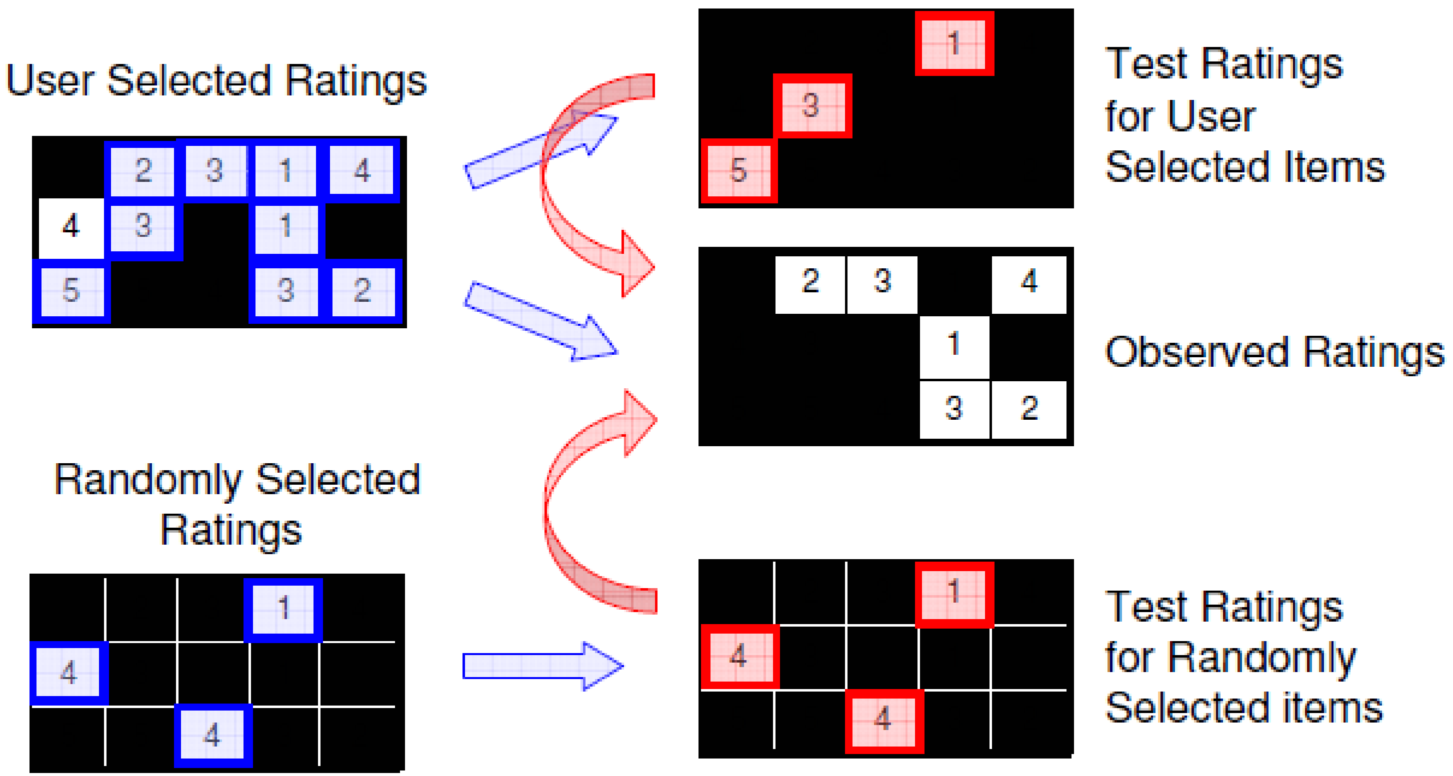
Strong Generalization

- Learn on training user observed ratings
- Evaluate on test user test ratings

Data Sets: User Splits

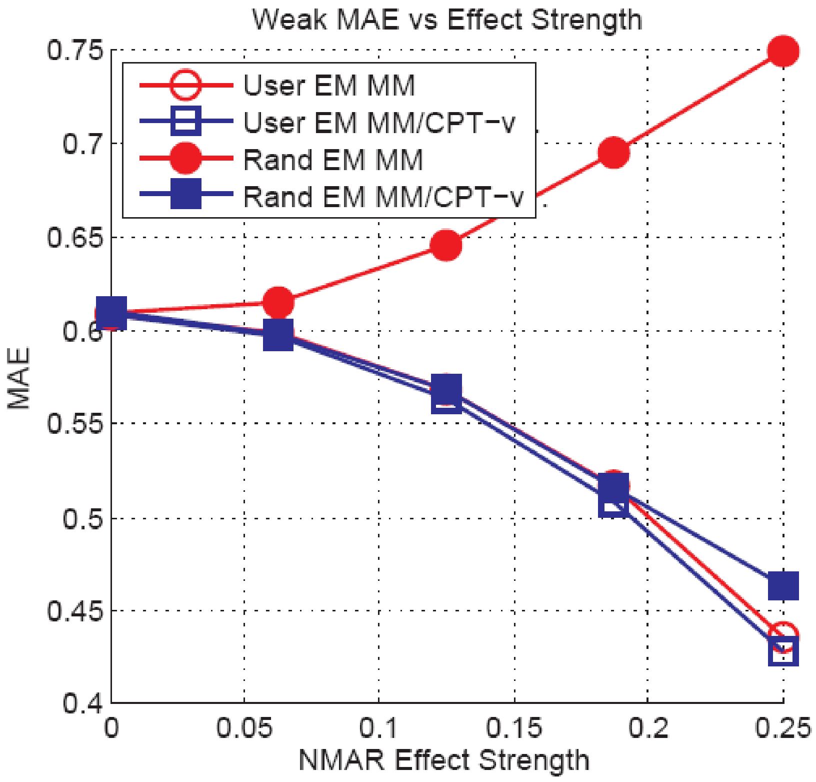


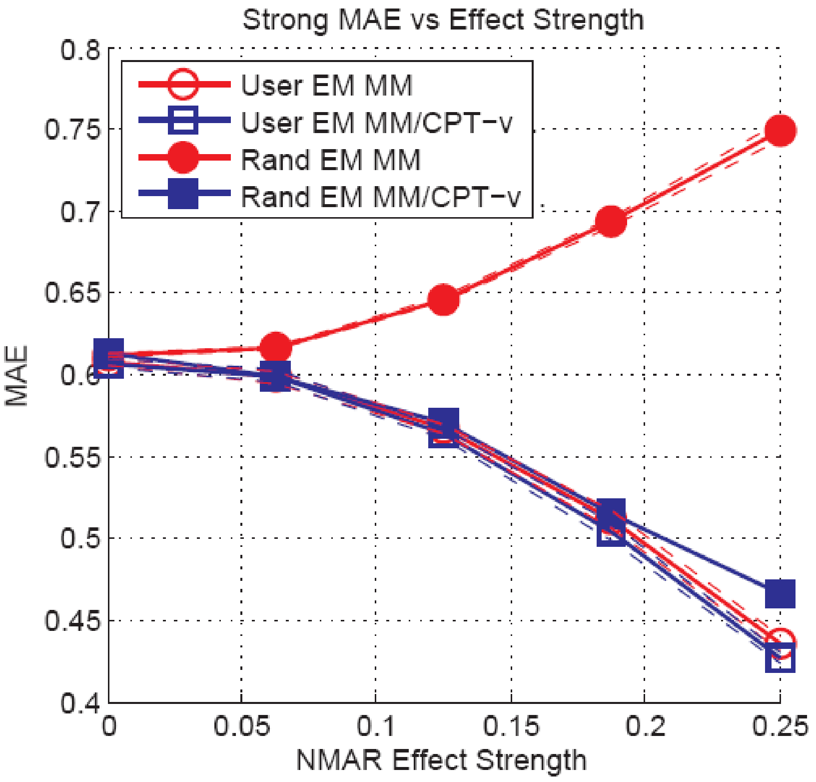
Data Sets: User Splits



Collaborative Filtering: Results

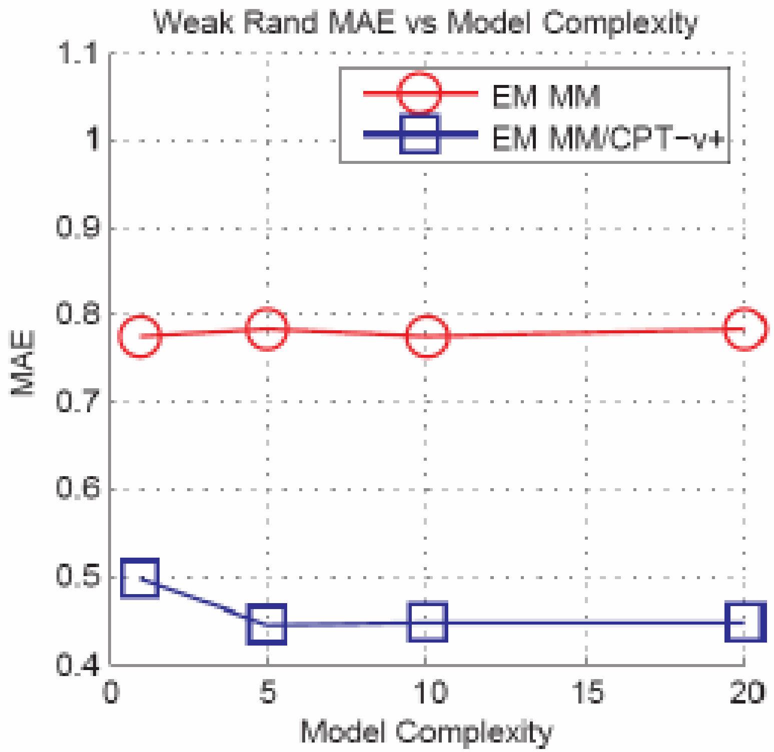
Jester Results: MM vs MM/CPT-v

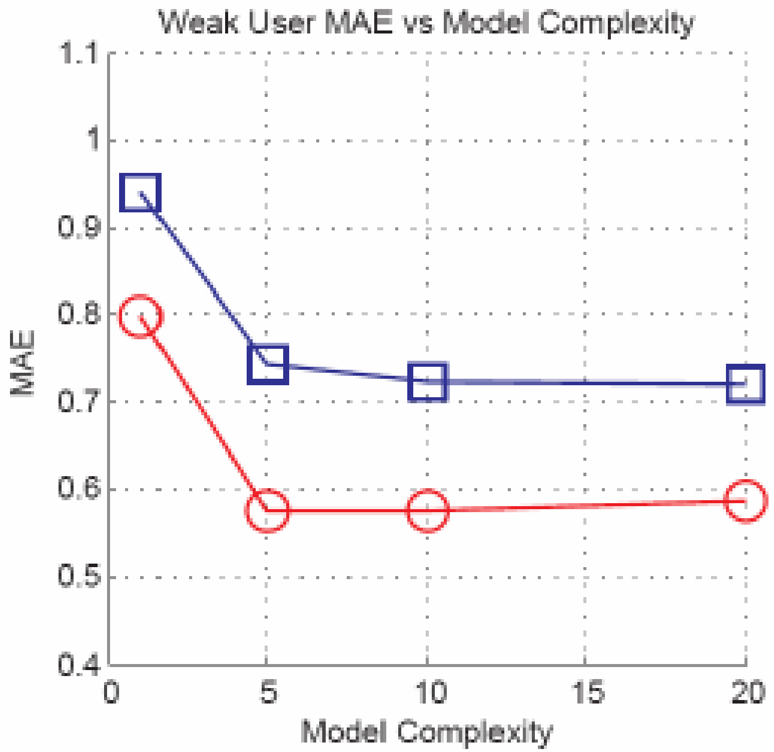




Collaborative Filtering: Results

Yahoo! Results: MM vs MM/CPT-v+





Collaborative Filtering: Results

Comparison of Results on Yahoo! Data

Method	Complexity	Rand MAE	User MAE
EM MM	1	0.7725 ± 0.0024	0.7626 ± 0.0077
EM MM/CPT-v	20	0.5431 ± 0.0012	0.6631 ± 0.0026
EM MM/Logit	5	0.5038 ± 0.0030	0.7029 ± 0.0186
EM MM/CPT-v+	5	0.4456 ± 0.0033	0.7235 ± 0.0059
MCMC DP	N/A	0.7624 ± 0.0063	0.5767 ± 0.0077
MCMC DP/CPT-v	N/A	0.5549 ± 0.0026	0.6670 ± 0.0071
MCMC DP/CPT-v+	N/A	0.4428 ± 0.0027	0.7537 ± 0.0026
CD cRBM	20	0.7179 ± 0.0025	0.5421 ± 0.0081
CD cRBM-v	1	0.4553 ± 0.0031	0.7501 ± 0.0066

Application to Ranking

$$NDCG(n) = \frac{\sum_{i=1}^{T} \frac{2^{x_{ni}^t} - 1}{\log(1 + \widehat{\pi}(i,n))}}{\sum_{i=1}^{T} \frac{2^{x_{ni}^t} - 1}{\log(1 + \pi(i,n))}}$$

- $\widehat{x}_{ni}^t$: mean of posterior predictive distribution for test item i.
- $\widehat{\pi}(i,n)$: rank of test item i according to $\widehat{x}_{ni}^t$.
- $\pi(i,n)$: rank of test item i according to x_{ni}^t .

Ranking Results