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The horizontal-vertical delusion

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Abstract. Most people can correctly apply the concepts of horizontal and vertical in describing objects, but a simple demonstration shows that they are confused about how these concepts work. The nature of the confusion and its possible causes are briefly discussed.

1 The demonstration

I am holding a large number of small aluminium rods. They are not aligned with one another. They are pointing in many different directions. I toss them all up in the air and they tumble and turn and bump into each other, so that their orientations are thoroughly randomised. Suddenly I stop time so that all the rods are frozen in place. Are there more that are roughly horizontal or more that are roughly vertical, or are the numbers about the same? By "roughly", I mean "within one degree of", and you can assume that physical factors like air resistance and the way the rods were thrown are irrelevant.

Please decide on your answer before continuing. Note your initial reaction as well as any subsequent reasoning.

Now try the second part of the demonstration. I am holding a large number of small aluminium disks. They are not aligned with one another. I toss them all up in the air and they tumble and turn and bump into each other, so that their orientations are thoroughly randomised. Suddenly I stop time so that all the disks are frozen in place. Are there more that are roughly horizontal or more that are roughly vertical, or are the numbers about the same?

Please decide on your answer before continuing. A lot of the point of the demonstration may be lost if you read the solution before attempting to decide on the answer for yourself.

2 The solution

I have tried both parts of this puzzle on a large number of people. When given the first part of the puzzle most people think that the numbers will be about the same. This is their initial reaction, and although they are suspicious of it, most people cannot find a reason to doubt it. They often justify their answer by saying that the orientations are random (and thus implying that there is no reason for preferring either horizontal or vertical). Actually, there are about 114 times as many rods that are within one degree of horizontal (the calculations are given below). The reason for the enormous discrepancy is that a horizontal rod can be rotated about a vertical axis and it will still be horizontal. It can also be rotated about its own axis and remain horizontal. For a vertical rod, only the rotation about its own axis preserves verticality. So horizontal leaves open two degrees of orientational freedom, whereas vertical only leaves open one.

I normally explained the solution to the first part of the puzzle before presenting the second part, so you may care to try the second part again before reading any further.

After misjudging the ratio by a factor of 114 in the first part of the demonstration, people were more cautious in this part and distributed their answers more evenly over the three possibilities. But despite being given the explanation for why there are more horizontal rods, many people still opted for "the same" when it came to disks. Some who correctly chose "more vertical" admitted that they did so for pragmatic reasons: they could tell that they were being set up. The easiest way to see the answer is to realise that the orientation of a disk can be represented by the orientation of its surface normal: every vertical disk has a horizontal surface normal, and every horizontal disk has a vertical surface normal. So the ratio is the same as before, but this time the vertical disks predominate.

3 The Gaussian sphere

To calculate the precise ratios, it is helpful to represent the orientation of a rod by a point on the surface of a sphere with a radius of 1. One end of the rod is translated to the centre of the sphere, and a point is then drawn on the surface where the rod (or its extension) intersects the surface. If the ends of the rod are not distinguished, the orientation can be represented by either of two diametrically opposite points on the surface of the sphere, depending on which end of the rod is translated to the centre of the sphere. The orientations of planes can be represented likewise by using their surface normals. People who are already familiar with the Gaussian sphere often give the correct answers to the demonstration task.

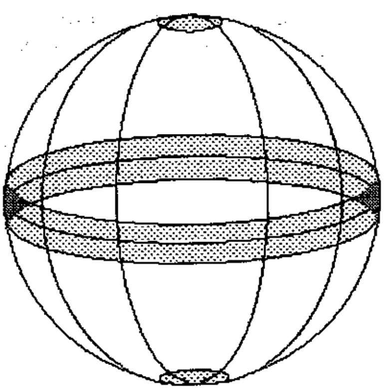

Figure 1. The Gaussian sphere with shaded areas showing the proportions of roughly vertical and roughly horizontal rods (see text for explanation).

Figure 1 shows points representing roughly vertical and roughly horizontal rods. The points that represent rods within one degree of vertical form two circles centred at the north and south poles with a radius equal to one degree measured in radians (which is $2\pi/360$). The area of these two circles is therefore approximately $2\pi(2\pi/360)^2$. This isn't quite exact because the circles are not quite flat. The points representing rods within one degree of horizontal form a band around the equator that has width $4\pi/360$ and length 2π. The area of this band is therefore approximately $8\pi^2/360$. So if points are uniformly distributed on the surface of the Gaussian sphere, the ratio of near-horizontal to near-vertical orientations is approximately $360/\pi \simeq 114$.

4 Why not use imagery?

Why don't people see that there are many times more rods which are nearly horizontal by forming an image and counting? With a ratio of 114 to 1, it should not be too difficult to notice the discrepancy. This question draws attention to the fact that generating an image is a complicated process. Decisions have to be made about what to generate. For many tasks, for example, it is best to generate an image of a two-dimensional structure (this doesn't necessarily mean that the image itself is two-dimensional). For the rods task, however, the natural two-dimensional image is fatal because it omits precisely the dimension in which the horizontal rods have an extra degree of freedom over the vertical ones.(1) In two dimensions, there really is a symmetry between horizontal and vertical which is lacking in three dimensions. One reason for the confidence with which many people give the wrong answer may be that they imagine a two-dimensional structure because they do not realise that omission of the third dimension has different effects on the two populations they are interested in. The apparent symmetry of horizontal and vertical not only suggests the wrong answer directly, it also biases the image that will be constructed to check the plausibility of the obvious answer.

In order to generate a 'fair' image it is necessary to have a uniform way of picking different orientations. It is not obvious that people can do this unless they use some representation like the Gaussian sphere. Imagery may seem to be useful for checking claims about the relative numbers of different kinds of things, but statistics gathered from the image are only as valid as the premises used to decide what to imagine.

5 Where does the confusion come from?

It is not at all obvious why people are so confused about horizontal and vertical. Normal adults in normal circumstances seldom make mistakes in applying the terms to three-dimensional objects. When shown a vertical surface, they seldom call it horizontal.(2) However, many people are convinced that the terms form a symmetrical pair

(1) Ian Howard has pointed out that some subjects interpret the question in terms of how the rods would appear *in projection* from a given vantage point. The correct answer to this question is that there are equal numbers of roughly horizontal and roughly vertical rod-images in a vertical picture plane. However, Howard points out that not all subjects interpret the question in terms of a two-dimensional projection, so the discussion given here is still relevant for the remaining subjects.

(2) After the second part of this demonstration one subject (a doctor with a PhD) held up a cigarette and starting at vertical she tilted it through about 60 degrees whilst saying "That's vertical, and that's vertical, and that's vertical, ...". Only when it was nearly horizontal did she realise that she had applied the term 'vertical' to a cigarette which was far from vertical. I can only conjecture that she had in mind that it lay in a vertical plane and was using the false but plausible inference that if a line lies in a vertical plane it must be vertical. After all, a line that lies in a horizontal plane must be horizontal. The dramatic deterioration in an existing competence as a result of an experience that challenges the subject's model is reminiscent of reports from the developmental literature.

and it is this false model that seems to lead to errors. Linguistic philosophers have argued that many philosophical problems arise because people have a false model of how the terms in their language work, and this seems to be just such a case. Being able to apply the terms 'horizontal' and 'vertical' correctly is sufficient for *describing* the world, but a model of how the terms work is necessary for *generating* an unbiased image or for reasoning about the categories. Part of this model, for many people, seems to be the belief that horizontal and vertical form a symmetrical pair. This belief would justify the conclusion that, in a random distribution, there would be as many vertical rods as horizontal ones.

But why should people have a false model of how the terms horizontal and vertical work? One reason, already mentioned, is that they *do* form a symmetrical pair in two dimensions. A second, more subtle, reason may be because the terms are used for both lines and planes in three dimensions. For lines, vertical is a much more special case than horizontal, but for planes it is the other way around. If we simply average over lines and planes the terms are symmetrical. Nobody who was consciously looking for regularities would deliberately blur the distinction between lines and planes, but the conscious pursuit of models may be a bad model of where most of our models come from.

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