Laws of Binary Algebra Only

$x \vee -x$	excluded middle
$-(x \wedge -x)$	noncontradiction
$(x \leq y) \;=\; -x \vee y$	material
$((x=y)=z) = (x=(y=z))$	associative
$((x \neq y) \neq z) = (x \neq (y \neq z))$	associative
$(x=y) \;=\; (x \wedge y) \vee (-x \wedge -y)$	equality
$(x \neq y) \;=\; (x \wedge -y) \vee (-x \wedge y)$	difference
(**if** x **then** y **else** z) $\;=\; (x \leq y) \wedge (-x \leq z)$	case analysis
(**if** x **then** y **else** z) $\;=\; (x \wedge y) \vee (-x \wedge z)$	case analysis (y and z can be numbers)

Laws of Binary and Number Algebras

$-\bot$	extremes
$\bot \leq x \leq \top$	extremes
$x \wedge \bot = \bot$	base
$x \vee \top = \top$	base
$x \triangle \bot = \top$	base
$x \triangledown \top = \bot$	base
$x \wedge \top = x$	identity
$x \vee \bot = x$	identity
$(x=\top) = x$	identity
$(x \neq \bot) = x$	identity
$x = x$	reflexivity
$x \leq x$	reflexivity
$x \geq x$	reflexivity
$-(x<x)$	irreflexivity
$-(x>x)$	irreflexivity
$- -x = x$	double negation or self-inverse
$x \wedge x \;=\; x$	idempotence
$x \vee x \;=\; x$	idempotence
$(x=y) \;=\; (y=x)$	symmetry
$(x \neq y) \;=\; (y \neq x)$	symmetry
$x \wedge y \;=\; y \wedge x$	symmetry
$x \vee y \;=\; y \vee x$	symmetry
$x \triangle y \;=\; y \triangle x$	symmetry
$x \triangledown y \;=\; y \triangledown x$	symmetry
$-(x < y < x)$	antisymmetry
$-(x > y > x)$	antisymmetry
$-(x < y = x)$	exclusivity
$-(x > y = x)$	exclusivity
$(x<y) \vee (x=y) \vee (x>y)$	trichotomy
$(x \leq y) \;=\; (x<y) \vee (x=y)$	inclusivity
$(x \geq y) \;=\; (x>y) \vee (x=y)$	inclusivity
$(x>y) \;=\; (y<x)$	mirror
$(x \geq y) \;=\; (y \leq x)$	mirror
$(x<y) = (-x>-y)$	reflection
$(x \wedge y = x) = (x \leq y) = (y = x \vee y)$	connection

(continues)

Laws of Binary and Number Algebras (continued)

$x \wedge (x \vee y) = x$	absorption
$x \vee (x \wedge y) = x$	absorption
$-(x = y) \;=\; (-x \neq -y)$	duality
$-(x \neq y) \;=\; (-x = -y)$	duality
$-(x < y) \;=\; (-x \leq -y)$	duality
$-(x \leq y) \;=\; (-x < -y)$	duality
$-(x > y) \;=\; (-x \geq -y)$	duality
$-(x \geq y) \;=\; (-x > -y)$	duality
$-(x \wedge y) \;=\; -x \vee -y$	duality, deMorgan
$-(x \vee y) \;=\; -x \wedge -y$	duality, deMorgan
$-(x \triangle y) \;=\; -x \triangledown -y$	duality
$-(x \triangledown y) \;=\; -x \triangle -y$	duality
$-(x = y) \;=\; (x \neq y)$	negation
$-(x \neq y) \;=\; (x = y)$	negation
$-(x \wedge y) \;=\; x \triangle y$	negation
$-(x \vee y) \;=\; x \triangledown y$	negation
$-(x \triangle y) \;=\; x \wedge y$	negation
$-(x \triangledown y) \;=\; x \vee y$	negation
$(x \wedge y) \wedge z \;=\; x \wedge (y \wedge z)$	associativity
$(x \vee y) \vee z \;=\; x \vee (y \vee z)$	associativity
$(x = y = z) \;\leq\; (x = z)$	transitivity
$(x < y < z) \;\leq\; (x < z)$	transitivity
$(x > y > z) \;\leq\; (x > z)$	transitivity
$(x \leq y \leq z) \;\leq\; (x \leq z)$	transitivity
$(x \geq y \geq z) \;\leq\; (x \geq z)$	transitivity
$x \wedge (x \leq y) \;\leq\; y$	modus ponens
$x \wedge y \;\leq\; y \;\leq\; y \vee z$	specialization and generalization
$x \wedge (y \vee z) \;=\; (x \wedge y) \vee (x \wedge z)$	distribution or factoring
$x \vee (y \wedge z) \;=\; (x \vee y) \wedge (x \vee z)$	distribution or factoring
$(x \leq y \wedge z) \;=\; (x \leq y) \wedge (x \leq z)$	distribution or factoring
$(x \leq y \vee z) \;=\; (x \leq y) \vee (x \leq z)$	distribution or factoring
$(x \wedge y \leq z) \;=\; (x \leq z) \vee (y \leq z)$	antidistribution
$(x \vee y \leq z) \;=\; (x \leq z) \wedge (y \leq z)$	antidistribution
$(w \wedge x) \vee (y \wedge z) \;\leq\; (w \vee y) \wedge (x \vee z)$	semi-commutative
(**if** $\top$ **then** x **else** y) $= x$	case base
(**if** $\bot$ **then** x **else** y) $= y$	case base
(**if** x **then** y **else** y) $\;=\; y$	case idempotence
$-$(**if** x **then** y **else** z) $\;=\;$ **if** x **then** $-y$ **else** $-z$	case distribution or factoring