

X7.4 Two competitors are playing a game. The score is kept by two natural implementer's variables  $A$  and  $B$ . The winner is reported to integer user's variable  $who$ . The operations are

$reset = A:=0. B:=0$

$Ascores = A:=A+1$

$Bscores = B:=B+1$

$whowon = \mathbf{if\ } A>B \mathbf{\ then\ } who:=1 \mathbf{\ else\ if\ } A=B \mathbf{\ then\ } who:=0 \mathbf{\ else\ } who:=-1 \mathbf{\ fi\ fi}$

Replace the two implementer's variables with one integer variable  $C$ .

- (a) What is the data transformer?
- (b) Transform all the operations.

After trying the question, scroll down to the solution.

(a) What is the data transformer?

$$\S \quad C = A - B$$

(b) Transform all the operations.

$$\begin{aligned} \S & \quad \forall A, B \cdot C = A - B \Rightarrow \exists A', B' \cdot C' = A' - B' \wedge \text{reset} \\ = & \quad \forall A, B \cdot C = A - B \Rightarrow \exists A', B' \cdot C' = A' - B' \wedge (A := 0 \cdot B := 0) \\ = & \quad \forall A, B \cdot C = A - B \Rightarrow \exists A', B' \cdot C' = A' - B' \wedge A' = 0 \wedge B' = 0 && \text{one-point twice} \\ = & \quad \forall A, B \cdot C = A - B \Rightarrow C' = 0 - 0 && \text{arithmetic} \\ = & \quad \forall A, B \cdot A = C + B \Rightarrow C' = 0 && \text{one-point} \\ = & \quad \forall B \cdot C' = 0 && \text{idempotent} \\ = & \quad C' = 0 \\ = & \quad C := 0 \\ & \quad \forall A, B \cdot C = A - B \Rightarrow \exists A', B' \cdot C' = A' - B' \wedge \text{Ascores} \\ = & \quad \forall A, B \cdot C = A - B \Rightarrow \exists A', B' \cdot C' = A' - B' \wedge (A := A + 1) \\ = & \quad \forall A, B \cdot C = A - B \Rightarrow \exists A', B' \cdot C' = A' - B' \wedge A' = A + 1 \wedge B' = B && \text{one-point twice} \\ = & \quad \forall A, B \cdot C = A - B \Rightarrow C' = A + 1 - B \\ = & \quad \forall A, B \cdot A = C + B \Rightarrow C' = A + 1 - B && \text{one-point} \\ = & \quad \forall B \cdot C' = C + B + 1 - B && \text{arithmetic} \\ = & \quad \forall B \cdot C' = C + 1 && \text{idempotent} \\ = & \quad C' = C + 1 \\ = & \quad C := C + 1 \\ & \quad \forall A, B \cdot C = A - B \Rightarrow \exists A', B' \cdot C' = A' - B' \wedge \text{Bscores} && \text{same} \\ = & \quad C := C - 1 \\ & \quad \forall A, B \cdot C = A - B \Rightarrow \exists A', B' \cdot C' = A' - B' \wedge \text{whowon} \\ = & \quad \forall A, B \cdot C = A - B \\ & \quad \Rightarrow \exists A', B' \cdot C' = A' - B' \\ & \quad \quad \wedge (\text{if } A > B \text{ then } \text{who} := 1 \\ & \quad \quad \quad \text{else if } A = B \text{ then } \text{who} := 0 \\ & \quad \quad \quad \text{else } \text{who} := -1 \text{ fi fi}) && \text{somehow} \\ = & \quad \text{if } C > 0 \text{ then } \text{who} := 1 \text{ else if } C = 0 \text{ then } \text{who} := 0 \text{ else } \text{who} := -1 \text{ fi fi} \end{aligned}$$