(unicorns) The following statements are made.  
All unicorns are white.  
All unicorns are black.  
No unicorn is both white and black.  
Are these statements consistent? What, if anything, can we conclude about unicorns?

§ Let \( \text{unicorn} \) be all unicorns. Let \( \text{white} \) and \( \text{black} \) be predicates on unicorns. Then

All unicorns are white:

(a) \( \forall u : \text{unicorn} \cdot \text{white } u \)

All unicorns are black:

(b) \( \forall u : \text{unicorn} \cdot \text{black } u \)

No unicorn is both white and black:

(c) \( \neg \exists u : \text{unicorn} \cdot \text{white } u \land \text{black } u \)

Suppose we take (a), (b), and (c) as axioms.

\[
\begin{align*}
\top & \quad \text{(a), (b), and (c) are axioms} \\
= & \quad (\forall u : \text{unicorn} \cdot \text{white } u) \land (\forall u : \text{unicorn} \cdot \text{black } u) \land (\neg \exists u : \text{unicorn} \cdot \text{white } u \land \text{black } u) \\
\text{Using a duality law (deMorgan) on (c), we can change it to a universal quantification:} \\
= & \quad (\forall u : \text{unicorn} \cdot \text{white } u) \land (\forall u : \text{unicorn} \cdot \text{black } u) \land (\forall u : \text{unicorn} \cdot \neg (\text{white } u \land \text{black } u)) \\
\text{Now we can use a splitting law to combine the three main conjuncts} \\
= & \quad \forall u : \text{unicorn} \cdot (\text{white } u \land \text{black } u) \land \neg (\text{white } u \land \text{black } u) \quad \text{Law of Noncontradiction} \\
= & \quad \forall u : \text{unicorn} \cdot \bot \quad \text{Law of Noncontradiction} \\
\text{Now suppose } U \text{ is a particular unicorn, } U : \text{unicorn} \ . \text{ Then by Specialization} \\
= & \quad \bot \\
\end{align*}
\]

So the statements are inconsistent under the assumption that \( U \) is a unicorn. But if \( \text{unicorn} = \text{null} \), then by the first quantifier law \( (\forall v : \text{null} \cdot b) \), all three statements are already theorems. If we are given (a), (b), and (c) as axioms, we must conclude that there are no unicorns.