Formalize the statements

Everyone loves my baby.
My baby loves only me.
I am my baby.

and prove that the first two statements imply the last statement.

After trying the question, scroll down to the solution.
\( \forall p \cdot (p \text{ loves mybaby}) \)

\((\text{mybaby loves me}) \land \neg (\exists p \cdot p \neq me \land (\text{mybaby loves } p))\)

\(me = \text{mybaby}\)

Now the proof: starting with the first two statements,

\((\forall p \cdot (p \text{ loves mybaby})) \land (\text{mybaby loves me}) \land \neg (\exists p \cdot p \neq me \land (\text{mybaby loves } p))\)

\(=\)

\((\forall p \cdot (p \text{ loves mybaby})) \land (\text{mybaby loves me}) \land (\forall p \cdot p = me \lor \neg (\text{mybaby loves } p))\)

specialize both \(p\)s

\(\Rightarrow\)

\((\text{mybaby loves mybaby}) \land (\text{mybaby loves me}) \land (\text{mybaby equals me} \lor \neg (\text{mybaby loves mybaby}))\)

use the first conjunct as context

\(=\)

\((\text{mybaby loves mybaby}) \land (\text{mybaby loves me}) \land (\text{mybaby equals me} \lor \neg \top)\)

binary axiom, base, and specialize

\(\Rightarrow\)

\(\text{mybaby equals me}\)

I suspect that “Everyone loves my baby.” should be formalized as

\(\forall p \cdot p \neq \text{mybaby} \Rightarrow (p \text{ loves mybaby})\)

and then the last statement is not provable from the first two.