Consider a fully parenthesized expression containing only the symbols $\top \bot = \oplus ( )$ in any quantity and any syntactically acceptable order.

§ The proofs will be by induction over the structure of the expressions. Every fully parenthesized expression containing only the symbols $\top \bot = \oplus ( )$ has one of the following four forms: $\top$, $\bot$, $(a=b)$, $(a \oplus b)$, where $a$ and $b$ are fully parenthesized expression containing only the symbols $\top \bot = \oplus ( )$.

(a) Show that all syntactically acceptable rearrangements are equivalent.

There are four alternatives. The first two alternatives are just a single symbol, so there are no rearrangements, so all zero rearrangements are equivalent. That's the base case. Now for the induction step.

Suppose the expression is $(a=b)$ for some expressions $a$ and $b$. The ways of rearranging $(a=b)$ are:

(a) rearrange $a$
(b) rearrange $b$
(c) change $(a=b)$ to $(b=a)$

First, consider (a). Make the inductive hypothesis that rearranging $a$ results in an expression that is equivalent to $a$. Then any expression with subexpression $a$ is equivalent to the same expression with subexpression $a$ replaced by its rearrangement. (This is formalized as the generic law of transparency.) Similarly for (b). For (c), we have the generic law of symmetry of $=$. That completes the proof for expressions of the form $(a=b)$.

Finally, suppose the expression is $(a+b)$ for some expressions $a$ and $b$. The ways of rearranging $(a+b)$ are:

(a) rearrange $a$
(b) rearrange $b$
(c) change $(a+b)$ to $(b+a)$

First, consider (a). Make the inductive hypothesis that rearranging $a$ results in an expression that is equivalent to $a$. Then any expression with subexpression $a$ is equivalent to the same expression with subexpression $a$ replaced by its rearrangement. (This is formalized as the generic law of transparency.) Similarly for (b). For (c), we have the generic law of symmetry of $\oplus$.

That completes the proof.

(b) Show that it is equivalent to any expression obtained from it by making an even number of the following substitutions: $\top$ for $\bot$, $\bot$ for $\top$, $=$ for $\oplus$, $\oplus$ for $=$.

§ Zero substitutions means the same expression, which is obviously equivalent. I will show that by making a single one of those substitutions, the expression is negated. Therefore two substitutions are a double negation, which is an equivalent expression. And so on for more substitutions.

If the expression is $\top$, the only substitution is $\bot$ for $\top$, and $\bot$ is the negation of $\top$.

If the expression is $\bot$, the only substitution is $\top$ for $\bot$, and $\top$ is the negation of $\bot$. 
Suppose the expression is $(a=b)$ for some expressions $a$ and $b$. The ways of making one substitution in $(a=b)$ are:

(a) make one substitution in $a$
(b) make one substitution in $b$
(c) change $(a=b)$ to $(a=b)$

First, consider (a). Make the inductive hypothesis that one substitutions in $a$ negates $a$, resulting in an expression equivalent to $(\neg a=b)$.

\[
\begin{align*}
\neg a = b & \quad \text{exclusion} \\
\equiv & \quad (a \neq b) \\
\equiv & \quad \neg (a = b)
\end{align*}
\]

so making one substitution in $a$ negates $(a=b)$. Similarly for (b). For (c),

\[
\begin{align*}
(a \neq b) & \quad \text{generic inequality} \\
\equiv & \quad \neg (a = b)
\end{align*}
\]

That completes the proof for expressions of the form $(a=b)$. Finally, suppose the expression is $(a\neq b)$ for some expressions $a$ and $b$. The ways of making one substitution in $(a\neq b)$ are:

(a) make one substitution in $a$
(b) make one substitution in $b$
(c) change $(a\neq b)$ to $(a=b)$

First, consider (a). Make the inductive hypothesis that one substitutions in $a$ negates $a$, resulting in an expression equivalent to $(\neg a\neq b)$.

\[
\begin{align*}
\neg a \neq b & \quad \text{generic inequality} \\
\equiv & \quad \neg (\neg a = b) \\
\equiv & \quad \neg (a \neq b)
\end{align*}
\]

so making one substitution in $a$ negates $(a\neq b)$. Similarly for (b). For (c),

\[
\begin{align*}
(a\neq b) & \quad \text{double negation} \\
\equiv & \quad \neg (a = b) \\
\equiv & \quad \neg (a\neq b)
\end{align*}
\]

That completes the proof.