Define \( \approx \) to give the deep contents of a string or set or list or function. Here are some examples.

\[
\approx(10; 11; 12; 13) = 10, 11, 12, 13 \\
\approx\{10, \{11, 12\}, 13\} = 10, 11, 12, 13 \\
\approx[10; \{11; 12\}; 13] = 10, 11, 12, 13 \\
\approx\{10, \{11, 12\}, 13\} = 10, 11, 12, 13 \\
\approx[10; \{11, 12\}; 13] = 10, 11, 12, 13 \\
\approx\langle x: \text{nat} \cdot \langle y: \text{nat} \cdot 2 \times x y \rangle \rangle = 2 \times \text{nat} \\
\approx\langle x: 0..4 \cdot \langle y: 0..x \cdot x + y \rangle \rangle = 1, 2, 3, 4, 5
\]

The contents operator \( \sim \) removes one level of structure from a set or list. The deep contents operator \( \approx \) removes all levels of structure.

After trying the question, scroll down to the solution.
§ It is convenient to start by defining $\approx$ for base cases. If $x$ is an element, and $x$ is not a set, string, list, or function, then

$$\approx x \equiv x$$

Now for bunches,

$$\approx \text{null} \equiv \text{null}$$
$$\approx (A, B) \equiv \approx A, \approx B$$

Now for sets,

$$\approx \{A\} \equiv \approx A$$

For strings,

$$\approx \text{null} \equiv \text{null}$$
$$\approx (A; B) \equiv \approx A, \approx B$$

For lists,

$$\approx [S] \equiv \approx S$$

For functions,

$$\approx \langle v: D \cdot e \rangle \equiv \langle v: D \cdot \approx e \rangle D$$

Or if you prefer,

$$\approx f \equiv \langle v: \Box f \cdot \approx f v \rangle (\Box f)$$

For functions we could instead define $\approx$ the way we usually define quantifiers.

$$\approx v: \text{null} \cdot e \equiv \text{null}$$
$$\approx v: x \cdot e \equiv \langle v: x \cdot \approx e \rangle x$$

for element $x$

$$\approx v: A, B \cdot e \equiv (\approx v: A \cdot e), (\approx v: B \cdot e)$$

$$\approx v: (\$v: D \cdot b) \cdot c \equiv \langle v: D \cdot \text{if } b \text{ then } \approx c \text{ else } \text{null} \rangle D$$