We have defined several quantifiers by starting with an associative symmetric operator with an identity. Bunch union is also such an operator. Does it yield a quantifier?

§ Yes. I'll use \( \mathcal{U} \) for the quantifier.

\[
\begin{align*}
\mathcal{U} v : \text{null} \cdot e &= \text{null} \\
\mathcal{U} v : x \cdot e &= \langle v : x \cdot e \rangle x \text{ for element } x \\
\mathcal{U} v : A, B \cdot e &= (\mathcal{U} v : A \cdot e), (\mathcal{U} v : B \cdot e) \\
\mathcal{U} v : (\$ v : D \cdot b) \cdot c &= \mathcal{U} v : D \cdot \text{if } b \text{ then } c \text{ else null } \text{fi}
\end{align*}
\]

Application of function \( f \) distributes over bunch union, so the \( \mathcal{U} \) quantifier gives the range of a function.

\( \mathcal{U} f = f(\Box f) \)