There are some people, some keys, and some doors. Let \( p \) holds \( k \) mean that person \( p \) holds key \( k \). Let \( k \) unlocks \( d \) mean that key \( k \) unlocks door \( d \). Let \( p \) opens \( d \) mean that person \( p \) can open door \( d \). Formalize

The solutions offered formalize one understanding of the sentences, but not the only reasonable understanding.

(a) Anyone can open any door if they have the appropriate key.
\[
\forall p:\ people \cdot \forall d:\ doors \cdot \forall k:\ keys \cdot (p \ opens \ d) \iff (p \ holds \ k) \land (k \ unlocks \ d)
\]

(b) At least one door can be opened without a key (by anyone).
\[
\exists d:\ doors \cdot \forall p:\ people \cdot p \ opens \ d \\
\text{or} \quad \exists d:\ doors \cdot \forall p:\ people \cdot \forall k:\ keys \cdot (p \ opens \ d) \iff \neg (p \ holds \ k) \land (k \ unlocks \ d)
\]

(c) The locksmith can open any door even without a key.
\[
\forall d:\ doors \cdot \text{locksmith} \ opens \ d
\]