Here are ten statements.

(i) Some criminal robbed the Russell mansion.
(ii) Whoever robbed the Russell mansion either had an accomplice among the servants or had to break in.
(iii) To break in one would have to either smash the door or pick the lock.
(iv) Only an expert locksmith could pick the lock.
(v) Anyone smashing the door would have been heard.
(vi) Nobody was heard.
(vii) No one could rob the Russell mansion without fooling the guard.
(viii) To fool the guard one must be a convincing actor.
(ix) No criminal could be both an expert locksmith and a convincing actor.
(x) Some criminal had an accomplice among the servants.

(a) Choosing good abbreviations, translate each of these statements into formal logic.

§ Here are some abbreviations.

\[ C_x \equiv (x \text{ is a criminal}) \]
\[ R_x \equiv (x \text{ robbed the Russell mansion}) \]
\[ S_x \equiv (x \text{ had an accomplice among the servants}) \]
\[ B_x \equiv (x \text{ broke in}) \]
\[ D_x \equiv (x \text{ smashed the door}) \]
\[ P_x \equiv (x \text{ picked the lock}) \]
\[ L_x \equiv (x \text{ is an expert locksmith}) \]
\[ H_x \equiv (x \text{ was heard}) \]
\[ F_x \equiv (x \text{ fooled the guard}) \]
\[ A_x \equiv (x \text{ is a convincing actor}) \]

Now the statements are formalized as follows.

(i) \[ \exists x \cdot C_x \land R_x \]
(ii) \[ \forall x \cdot R_x \Rightarrow S_x \lor B_x \]
(iii) \[ \forall x \cdot B_x \Rightarrow D_x \lor P_x \]
(iv) \[ \forall x \cdot L_x \Leftarrow P_x \]
(v) \[ \forall x \cdot D_x \Rightarrow H_x \]
(vi) \[ \neg \exists x \cdot H_x \]
(vii) \[ \neg \exists x \cdot R_x \land \neg F_x \]
(viii) \[ \forall x \cdot F_x \Rightarrow A_x \]
(ix) \[ \neg \exists x \cdot C_x \land L_x \land A_x \]
(x) \[ \exists x \cdot C_x \land S_x \]

(b) Taking the first nine statements as axioms, prove the tenth.

§ Lemma:
\[ \top \]
\[ \equiv \neg \exists x \cdot R_x \land \neg F_x \]
\[ \equiv \forall x \cdot \neg (R_x \land \neg F_x) \]
\[ \equiv \forall x \cdot \neg R_x \lor \neg F_x \]
\[ \equiv \forall x \cdot \neg R_x \land F_x \]
\[ \equiv \forall x \cdot R_x \Rightarrow F_x \]

Now the main proof:
\[ \top \]
\[ \equiv \exists x \cdot C_x \land R_x \]
\[ \equiv \exists x \cdot C_x \land R_x \land R_x \]
\[ \Rightarrow \exists x \cdot C_x \land F_x \land (S_x \lor B_x) \]
\[ \Rightarrow \exists x \cdot C_x \land A_x \land (S_x \lor D_x \lor P_x) \]
\[ \Rightarrow \exists x \, C_x \land A_x \land (S_x \lor H_x \lor L_x) \quad \text{distribute} \]
\[ \equiv \exists x \, C_x \land A_x \land S_x \lor C_x \land A_x \land H_x \lor C_x \land A_x \land L_x \quad \text{(vi) and (ix)} \]
\[ \equiv \exists x \, C_x \land A_x \land S_x \quad \text{specialize} \]
\[ \Rightarrow \exists x \, C_x \land S_x \]