

78 (liar) Epimenides was a Cretan who said “All Cretans are liars.”; this has become known as the liar's paradox. Formalize and analyze this sentence.

After trying the question, scroll down to the solution.

§ Epimenides intended his statement to be an example of self-contradiction, or inconsistency. Saint Paul missed the point completely, taking Epimenides' statement at face value, and elaborating: “It was one of themselves, one of their own prophets, who said, “Cretans were never anything but liars, dangerous animals, and lazy”: and that is a true statement.” [Jerusalem Bible, Reader's Edition, Titus, chapter 1 verse 12].

A liar is defined as someone who sometimes (but not necessarily always) lies. But perhaps Epimenides meant that a liar is someone who always lies. I'll look at both meanings. Let *Cretans* be the bunch of all Cretans. For each *c* in *Cretans*, let *S c* be the bunch of all statements made by *c*.

Suppose a liar is someone who sometimes lies. Then “All Cretans are liars.” becomes

$$\forall c: Cretans. \exists m: S c. \neg m$$

This statement could be true; if it is, then Epimenides said a true statement, but Epimenides, like all Cretans, sometimes says falsehoods. Or the statement could be false; if it is, then somewhere there's a Cretan (but not Epimenides) who always tells the truth. With this meaning for “liar”, there's no self-contradiction, no inconsistency, no paradox.

Now suppose a liar is someone who always lies. Then “All Cretans are liars.” becomes

$$\forall c: Cretans. \forall m: S c. \neg m$$

Let's call this statement *A*. Let *E* be Epimenides. Then

$$\begin{aligned} & A && \text{definition of } A \\ = & \forall c: Cretans. \forall m: S c. \neg m && \text{specialization } E: Cretans \\ \Rightarrow & \forall m: S E. \neg m && \text{specialization } A: S E \\ \Rightarrow & \neg A \end{aligned}$$

We have proven $A \Rightarrow \neg A$.

$$\begin{aligned} & \top && \text{just proven} \\ = & A \Rightarrow \neg A && \text{inclusion (material implication)} \\ = & \neg A \vee \neg A && \text{idempotence} \\ = & \neg A \end{aligned}$$

and so we have proven $\neg A$, or $A = \perp$. So the statement “All Cretans are liars.” is false. Epimenides is saying something false, but at another time he might say something true, or some other Cretan might say something true. With this meaning for “liar”, there's still no self-contradiction, no inconsistency, no paradox; just a false statement.

Let *L* be the statement “This statement is false.”. Formalizing, the statement is simply

$$L = \perp$$

and since that is statement *L*,

$$L = (L = \perp)$$

This equation simplifies to \perp and has no solution for *L*. The equation $L = \perp$ can be neither \top nor \perp . The statement “This statement is false.” is a self-contradiction, or inconsistency, or paradox. See Exercise [3](#).