77 (sick) John says "If one of us gets sick, then we all get sick.". Mary disagrees. She says "If one of us stays well, then we all stay well.". Settle the argument.

After trying the question, scroll down to the solution.

§ I formalize " x gets sick" as *sick x* . I take " x stays well" to be the negation of " x gets sick". Quantifying over "us", Mary says

    $(\exists x\cdot \neg\ sick\ x) \Rightarrow (\forall x\cdot \neg\ sick\ x)$ quantifier duality twice

$=$ $\neg(\forall x\cdot\ sick\ x) \Rightarrow \neg(\exists x\cdot\ sick\ x)$ exclusion

$=$ $(\exists x\cdot\ sick\ x) \Rightarrow \neg\ \neg(\forall x\cdot\ sick\ x)$ double negation

$=$ $(\exists x\cdot\ sick\ x) \Rightarrow (\forall x\cdot\ sick\ x)$

and that's exactly what John says. So they agree. There's no argument.