Express formally that \( L \) is a longest sorted sublist of \( M \) where

Let \( T \) be the type of item in the lists.

(a) the sublist must be consecutive items (a segment).

Define relation \( S \) so that \( S L M \) says that list \( L \) is a sorted segment of list \( M \) as follows:

\[
S = \langle L, M: [\star T] \rightarrow 0 \leq i < j \leq \#L \leq \#M \land L = M[\ldots i;..j] \land \forall k, l: i \leq k \leq l \leq j \Rightarrow M_k \leq M_l \rangle
\]

The answer is \( S L M \land \neg \exists K: [\star T]: S K M \land \#K > \#L \).

This question can be interpreted differently. It might mean that \( L \) is a sorted segment of \( M \) that cannot be extended on either end to be a longer sorted segment. In other words, that it is locally longest, rather than globally longest.

(b) the sublist must be consecutive (a segment) and nonempty.

Define relation \( S \) so that \( S L M \) says that list \( L \) is a sorted nonempty segment of list \( M \) as follows (\( T \) is the type of item in the lists):

\[
S = \langle L, M: [\star T] \rightarrow 0 \leq i < j \leq \#L \leq \#M \land L = M[\ldots i;..j] \land \forall k, l: i \leq k \leq l \leq j \Rightarrow M_k \leq M_l \rangle
\]

The answer is \( S L M \land \neg \exists K: [\star T]: S K M \land \#K > \#L \).

(c) the sublist contains items in their order of appearance in \( M \), but not necessarily consecutively (not necessarily a segment).

Define (domains are lists)

\[
S = \langle L, M: [\star T] \rightarrow \#L = 0 \lor \exists i: \square M \cdot L_0 = M i \land S(L[1;\ldots L]) (M[i+1;\ldots \#M]))
\]

so \( S L M \) means that \( L \) is a sublist of \( M \) with items in the same order but not necessarily consecutively. Then the desired expression is

\[
S L M \land \neg \exists K: [\star T]: S K M \land \#K > \#L
\]

Another solution might be

\[
\exists N: [\star T]: \#N = \#L \land \sum N + \#N \leq \#M \land \forall i: \square L \cdot L_i = M((\sum N[0;\ldots i+1])+i)
\]

but I'm not sure.