There are some people, some keys, and some doors. Let \( p \text{ holds } k \) mean that person \( p \) holds key \( k \). Let \( k \text{ unlocks } d \) mean that key \( k \) unlocks door \( d \). Let \( p \text{ opens } d \) mean that person \( p \) can open door \( d \). Formalize

The solutions offered formalize one understanding of the sentences, but not the only reasonable understanding.

(a) Anyone can open any door if they have the appropriate key.
\[
\forall p: \text{people} \cdot \forall d: \text{doors} \cdot \forall k: \text{keys} \cdot (p \text{ opens } d) \iff (p \text{ holds } k) \land (k \text{ unlocks } d)
\]

(b) At least one door can be opened without a key (by anyone).
\[
\exists d: \text{doors} \cdot \forall p: \text{people} \cdot p \text{ opens } d
\]
or
\[
\exists d: \text{doors} \cdot \forall p: \text{people} \cdot \forall k: \text{keys} \cdot (p \text{ opens } d) \iff \neg (p \text{ holds } k) \land (k \text{ unlocks } d)
\]

(c) The locksmith can open any door even without a key.
\[
\forall d: \text{doors} \cdot \text{locksmith opens } d
\]