(arity) The arity of a function is the number of variables (parameters) it introduces, and the number of arguments it can be applied to. Write axioms to define \( \alpha f \) (arity of \( f \)).

§ As a first effort, we might try

\[
\begin{align*}
\alpha x &= 0 \quad \text{if } x \text{ is not a function} \\
\alpha (A \rightarrow B) &= 1 + \alpha B 
\end{align*}
\]

Unfortunately, \( \text{null} \rightarrow \text{null} \rightarrow 3 = \text{null} \rightarrow 3 \), so we would prove \( 2 = 1 \). And there are functions of mixed arity (similar to the list \([2; [3]]\); if indexed by 0 it cannot be indexed again, but if indexed by 1 it can be indexed again). So we try again.

\[
\begin{align*}
\alpha x &= 0 \quad \text{if } x \text{ is not a function} \\
\alpha f &= 1 + \alpha(f(\Box f)) \quad \text{if } f \text{ is a function}
\end{align*}
\]