Express formally that \( L \) is a longest sorted sublist of \( M \) where

(a) the sublist must be consecutive items (a segment).
§ Define relation \( S \) so that \( S L M \) says that list \( L \) is a sorted segment of list \( M \) as follows (\( T \) is the type of item in the lists):
\[
S = \langle L, M : [*T] \rightarrow \exists i, j : \text{nat} \cdot 0 \leq i \leq j \leq \#L \wedge L = M[i..j] \wedge \forall k, l \cdot i \leq k \leq l \leq j \Rightarrow L_k \leq L_l \rangle
\]
The answer is \( S L M \wedge \neg \exists K : [*T] \cdot S K M \wedge \#K > \#L \).
This question can be interpreted differently. It might mean that \( L \) is a sorted segment of \( M \) that cannot be extended on either end to be a longer sorted segment. In other words, that it is locally longest, rather than globally longest.

(b) the sublist must be consecutive (a segment) and nonempty.
§ Define relation \( S \) so that \( S L M \) says that list \( L \) is a sorted nonempty segment of list \( M \) as follows (\( T \) is the type of item in the lists):
\[
S = \langle L, M : [*T] \rightarrow \exists i, j : \text{nat} \cdot 0 < i < j \leq \#L \leq \#M \wedge L = M[i..j] \wedge \forall k, l \cdot i \leq k \leq l \leq j \Rightarrow L_k \leq L_l \rangle
\]
The answer is \( S L M \wedge \neg \exists K : [*T] \cdot S K M \wedge \#K > \#L \).

(c) the sublist contains items in their order of appearance in \( M \), but not necessarily consecutively (not necessarily a segment).
§ \( \exists N : [*T] \cdot \#N = \#L \wedge \sum N + \#N \leq \#M \wedge \forall i : 0..\#L \cdot L_i = M(\sum N[0..i+1]+i) \)
Another solution is to define (domains are lists)
\[
S = \langle L, M : [*T] \rightarrow \#L = 0 \vee \exists i : 0..\#M \cdot L_0 = BM_i \wedge S (L[1..\#L]) (M[i+1..\#M]) \rangle
\]
so \( S L M \) means that \( L \) is a sublist of \( M \) with items in the same order but not necessarily consecutively. Then the desired expression is
\[
S L M \wedge \neg \exists K : [*T] \cdot S K M \wedge \#K > \#L \]