Express formally that \( L \) is a longest sorted sublist of \( M \) where

Let \( T \) be the type of item in the lists.

(a) the sublist must be consecutive items (a segment).

Define relation \( S \) so that \( S \subseteq L \subseteq M \) says that list \( L \) is a sorted segment of list \( M \) as follows:

\[
S = \langle L, M : [*T] \rightarrow \exists i, j : 0 \leq i < j \leq \#L \leq \#M \land L[M[i..j]] \land \forall k, l : i \leq k \leq l \leq j \Rightarrow M_k \leq M_l \rangle
\]

The answer is \( S \subseteq L \subseteq M \land \neg \exists K : [*T] \cdot S \subseteq K \subseteq M \land \#K > \#L \).

This question can be interpreted differently. It might mean that \( L \) is a sorted segment of \( M \) that cannot be extended on either end to be a longer sorted segment. In other words, that it is locally longest, rather than globally longest.

(b) the sublist must be consecutive (a segment) and nonempty.

Define relation \( S \) so that \( S \subseteq L \subseteq M \) says that list \( L \) is a sorted nonempty segment of list \( M \) as follows (\( T \) is the type of item in the lists):

\[
S = \langle L, M : [*T] \rightarrow \exists i, j : 0 < i < j \leq \#L \leq \#M \land L[M[i..j]] \land \forall k, l : i \leq k \leq l \leq j \Rightarrow M_k \leq M_l \rangle
\]

The answer is \( S \subseteq L \subseteq M \land \neg \exists K : [*T] \cdot S \subseteq K \subseteq M \land \#K > \#L \).

(c) the sublist contains items in their order of appearance in \( M \), but not necessarily consecutively (not necessarily a segment).

Define (domains are lists)

\[
S = \langle L, M : [*T] \rightarrow \#L = 0 \lor \exists i : 0..\#M \cdot L[0] = M_i \land S(L[i+1..\#M]) \rangle
\]

so \( S \subseteq L \subseteq M \) means that \( L \) is a sublist of \( M \) with items in the same order but not necessarily consecutively. Then the desired expression is

\[
S \subseteq L \subseteq M \land \neg \exists K : [*T] \cdot S \subseteq K \subseteq M \land \#K > \#L
\]

Another solution might be

\[
\exists N : [*T] \cdot \#N = \#L \land \sum \#N \leq \#M \land \forall i : 0..\#L \cdot L_i = M(\sum \#N[i]; i + 1) + i
\]

but I'm not sure.