Implementer's variables $p, q$: real represent two points along a line. Each number tells the distance of one point from the origin (a standard point). They must be reimplemented by one implementer's variable $r$: real which tells the distance from $p$ to $q$. For examples, if $p=3$ and $q=5$, then $r=2$; if $p=5$ and $q=3$, then $r=-2$.

(a) What is the data transformer?

§

$r = q - p$

(b) A user has binary variable $b$ and operation

\[ \text{compare} \equiv b := q \geq p \]

Use your transformer from part (a) to transform operation compare.

§

$\forall p, q \cdot r = q - p \Rightarrow \exists p', q' \cdot r' = q' - p' \land (b := q \geq p)$ expand assignment

$\equiv \forall p, q \cdot r = q - p \Rightarrow r' = q - p \land b' = q \geq p$ one-point twice

$\equiv \forall p, q \cdot r = q - p \Rightarrow r' = q - p \land b' = q \geq p$ context

$\equiv \forall p, q \cdot r = q - p \Rightarrow r' = r \land b' = r \geq 0$ some law of arithmetic

$\equiv \forall p, q \cdot p = q - r \Rightarrow r' = r \land b' = r \geq 0$ one-point and idempotent

$\equiv r' = r \land b' = r \geq 0$ definition of assignment

$\equiv b := r \geq 0$