

- 69 In each of the following, replace p by
 $\langle x: \text{int} \cdot \langle y: \text{int} \cdot \langle z: \text{int} \cdot x \geq 0 \wedge x^2 \leq y \wedge \forall z: \text{int} \cdot z^2 \leq y \Rightarrow z \leq x \rangle \rangle \rangle$
and simplify, assuming $x, y, z, u, w: \text{int}$.
- (a) $p(x+y)(2 \times u + w)z$
(b) $p(x+y)(2 \times u + w)$
(c) $p(x+z)(y+y)(2+z)$

After trying the question, scroll down to the solution.

(a) $p(x+y)(2xu+w)z$
 § $\langle x: \text{int} \cdot \langle y: \text{int} \cdot \langle z: \text{int} \cdot x \geq 0 \wedge x^2 \leq y \wedge \forall z: \text{int} \cdot z^2 \leq y \Rightarrow z \leq x \rangle \rangle \rangle (x+y)(2xu+w)z$
 Variables x , y , and z appear both locally and nonlocally.
 Variable z is introduced twice locally.

To avoid confusion, I will rename the local variables to a , b , c , and d .
 (Since the first local z is unused, I don't need to rename it to c , but I will anyway.)

$$= \langle a: \text{int} \cdot \langle b: \text{int} \cdot \langle c: \text{int} \cdot a \geq 0 \wedge a^2 \leq b \wedge \forall d: \text{int} \cdot d^2 \leq b \Rightarrow d \leq a \rangle \rangle \rangle (x+y)(2xu+w)z$$

apply 3 times

$$= x+y \geq 0 \wedge (x+y)^2 \leq 2xu+w \wedge \forall d: \text{int} \cdot d^2 \leq 2xu+w \Rightarrow d \leq x+y$$

(b) $p(x+y)(2xu+w)$
 § $\langle x: \text{int} \cdot \langle y: \text{int} \cdot \langle z: \text{int} \cdot x \geq 0 \wedge x^2 \leq y \wedge \forall z: \text{int} \cdot z^2 \leq y \Rightarrow z \leq x \rangle \rangle \rangle (x+y)(2xu+w)$
 Variables x , y , and z appear both locally and nonlocally.
 Variable z is introduced twice locally.

To avoid confusion, I will rename the local variables to a , b , c , and d .
 (Since the first local z is unused, I don't need to rename it to c , but I will anyway.)

$$= \langle a: \text{int} \cdot \langle b: \text{int} \cdot \langle c: \text{int} \cdot a \geq 0 \wedge a^2 \leq b \wedge \forall d: \text{int} \cdot d^2 \leq b \Rightarrow d \leq a \rangle \rangle \rangle (x+y)(2xu+w)$$

apply 2 times

$$= \langle c: \text{int} \cdot x+y \geq 0 \wedge (x+y)^2 \leq 2xu+w \wedge \forall d: \text{int} \cdot d^2 \leq 2xu+w \Rightarrow d \leq x+y \rangle$$

note that c is unused.

$$= \text{int} \rightarrow (x+y \geq 0 \wedge (x+y)^2 \leq 2xu+w \wedge \forall d: \text{int} \cdot d^2 \leq 2xu+w \Rightarrow d \leq x+y)$$

(c) $p(x+z)(y+y)(2+z)$
 § $\langle x: \text{int} \cdot \langle y: \text{int} \cdot \langle z: \text{int} \cdot x \geq 0 \wedge x^2 \leq y \wedge \forall z: \text{int} \cdot z^2 \leq y \Rightarrow z \leq x \rangle \rangle \rangle (x+z)(y+y)(2+z)$
 Variables x , y , and z appear both locally and nonlocally.
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To avoid confusion, I will rename the local variables to a , b , c , and d .
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$$= \langle a: \text{int} \cdot \langle b: \text{int} \cdot \langle c: \text{int} \cdot a \geq 0 \wedge a^2 \leq b \wedge \forall d: \text{int} \cdot d^2 \leq b \Rightarrow d \leq a \rangle \rangle \rangle (x+z)(y+y)(2+z)$$

apply 3 times

$$= x+z \geq 0 \wedge (x+z)^2 \leq y+y \wedge \forall d: \text{int} \cdot d^2 \leq y+y \Rightarrow d \leq x+z$$