Simplify (no proof)

(a) \[0 \rightarrow 1 | 1 \rightarrow 2 | 2 \rightarrow 3 | 3 \rightarrow 4 | 4 \rightarrow 5 | [0;..5]
\]
§ \[0 \rightarrow 1 | 1 \rightarrow 2 | 2 \rightarrow 3 | 3 \rightarrow 4 | 4 \rightarrow 5 | [0; 1; 2; 3; 4]
\]
\[= 0 \rightarrow 1 | 1 \rightarrow 2 | 2 \rightarrow 3 | 3 \rightarrow 4 | [0; 1; 2; 3; 5]
\]
\[= 0 \rightarrow 1 | 1 \rightarrow 2 | 2 \rightarrow 3 | [0; 1; 2; 4; 5]
\]
\[= 0 \rightarrow 1 | [0; 2; 3; 4; 5]
\]
\[= [1; 2; 3; 4; 5]
\]

(b) \[(4 \rightarrow 2 | [-3;..3]) 3
\]
§ \[(4 \rightarrow 2 | [-3;..3]) 3
\]
\[= (4 \rightarrow 2 | [-3; -2; -1; 0; 1; 2]) 3
\]
\[= [-3; -2; -1; 0; 2; 2] 3
\]
\[= 0
\]

(c) \[((3;2) \rightarrow [10;..15] | 3 \rightarrow [5;..10] | [0;..5]) 3
\]
§ \[((3;2) \rightarrow [10;..15] | 3 \rightarrow [5;..10] | [0;..5]) 3
\]
\[= ((3;2) \rightarrow [10;..15] | 3 \rightarrow [5;..10] | [0; 1; 2; 3; 4]) 3
\]
\[= ((3;2) \rightarrow [10;..15] | [0; 1; 2; 5;..10]; 4) 3
\]
\[= ([0; 1; 2; 5; 6; [10;..15]; 8; 9]; 4) 3
\]
\[= [5; 6; [10;..15]; 8; 9]
\]

(d) \[(0;..5) [3; 4]) 1
\]
§ One way:
\[(0;..5) [3; 4]) 1
\]
\[= ([0;..5] 3; [0;..5] 4) 1
\]
\[= [0;..5] 4
\]
\[= 4
\]
Another way:
\[(0;..5) [3; 4]) 1
\]
\[= [0;..5] (3; 4] 1
\]
\[= [0;..5] 4
\]
\[= 4
\]

(e) \[(2;2) \rightarrow “j” | [“abc”]; [“de”]; [“fghi”]
\]
§ Item 2 of [“abc”]; [“de”]; [“fghi”] is [“fghi”] and its item 2 is “h” so replacing item 2:2 or [“abc”]; [“de”]; [“fghi”] with “j” gives [“abc”]; [“de”]; [“fgji”]

(f) \# [nat] 1 because “A nonempty bunch of items is also an item.” page 17
§ or, informally
\#[nat]
\[= #([0, 1, 2, 3, ...]
\]
\[= #([0], [1], [2], [3], ...)
\]
\[= #([0], #1, #2, #3, ...)
\]
\[= 1, 1, 1, 1, ...
\]
\[= 1
This is the sort of “proof” that mathematicians accept, but it's not a formal proof because the three dots mean “guess what goes here”. Anyway, the question did not ask for proof.

\[(g)\]
\[\#[*3]\]
\[\#[*3] \]
\[= \#[\text{nil, 3, 3;3, 3;3;3, ...}]\]
\[= \#([\text{nil}, [3], [3;3], [3;3;3], ...])\]
\[= \#([\text{nil}, #[3], #[3;3], #[3;3;3], ...]\]
\[= 0, 1, 2, 3, ...\]
\[= \text{nat}\]
Again, an informal “proof”, but the question did not ask for proof.

\[(h)\]
\[[3; 4]: [3*4*int]\]
\[\#4*int = int; int; int; int\]
\[3*4*int = int; int; int; int; int; int; int; int; int; int\]
\[[3*4*int] = [int; int; int; int; int; int; int; int; int; int]\]
which is all lists of 12 integers, and [3; 4] is not a list of 12 integers, so the answer is \[\bot\]

\[(i)\]
\[[3; 4]: [3; int]\]
\[\#3; int\] is all lists of length 2 whose item 0 is 3 and whose item 1 is in int. The list [3; 4] is one of them, so the answer is \[\top\]

\[(j)\]
\[[3, 4; 5]: [2*int]\]
\[\#2*int\] is all lists of length 2 whose item whose items are both in int.
\[[3, 4; 5]\]
\[= [3, (4; 5)]\]
\[= [3], [4; 5]\]
and [3] is not a list of length 2, so the answer is \[\bot\]

\[(k)\]
\[[3, 4; 5]: [2*int]\]
\[\#(3, 4; 5) = [3; 5, 4; 5] = [3; 5], [4; 5]\]
and both these lists are of length 2 and both items of each are in int so the answer is \[\top\]

\[(l)\]
\[[3; (4, 5); 6; (7, 8, 9)] \# [3; 4; (5, 6); (7, 8)]\]
\[\# [3; (4, 5); 6; (7, 8, 9)] \# [3; 4; (5, 6); (7, 8)]\]
\[= ([3; 4; 6; 7], [3; 5; 6; 7], [3; 4; 6; 8], [3; 5; 6; 8], [3; 4; 6; 9], [3; 5; 6; 9]) \]
\[\# (3; 4; 5; 7), [3; 4; 6; 7], [3; 4; 5; 8], [3; 4; 6; 8])\]
\[= [3; 4; 6; 7], [3; 4; 6; 8]\]
\[= [3; 4; 6; (7, 8)]\]