Simplify (no proof)

(a) 0 → 1 | 1 → 2 | 2 → 3 | 3 → 4 | 4 → 5 | [0;..5]
§ 0 → 1 | 1 → 2 | 2 → 3 | 3 → 4 | 4 → 5 | [0; 1; 2; 3; 4]
   = 0 → 1 | 1 → 2 | 2 → 3 | 3 → 4 | [0; 1; 2; 3; 5]
   = 0 → 1 | 1 → 2 | 2 → 3 | [0; 1; 2; 4; 5]
   = 0 → 1 | [0; 2; 3; 4; 5]
   = [1; 2; 3; 4; 5]
   = [1;..6]

(b) (4 → 2 | [–3;..3]) 3
§ (4 → 2 | [–3;..3]) 3
   = (4 → 2 | [–3; –2; –1; 0; 1; 2]) 3
   = [–3; –2; –1; 0; 2; 2] 3
   = 0

(c) ((3;2) → [10;..15] | 3 → [5;..10] | [0;..5]) 3
§ ((3;2) → [10;..15] | 3 → [5;..10] | [0;..5]) 3
   = ((3;2) → [10;..15] | 3 → [5;..10] | [0; 1; 2; 3; 4]) 3
   = ((3;2) → [10;..15] | [0; 1; 2; [5;..10]; 4]) 3
   = ((3;2) → [10;..15] | [0; 1; 2; [5; 6; 7; 8; 9]; 4]) 3
   = [0; 1; 2; 5; 6; [10;..15]; 8; 9]; 4] 3
   = [5; 6; [10;..15]; 8; 9]

(d) ([0;..5] [3; 4]) 1
§ One way:
   ([0;..5] [3; 4]) 1
   = [[0;..5] 3; [0;..5] 4] 1
   = [0;..5] 4
   = 4

Another way:
   ([0;..5] [3; 4]) 1
   = [0;..5] ([3; 4] 1)
   = [0;..5] 4
   = 4

(e) (2;2) → “j” | [“abc”]; [“de”]; [“fghi”]
§ Item 2 of [“abc”]; [“de”]; [“fghi”] is [“fghi”] and its item 2 is “h” so replacing item 2:2 or [“abc”]; [“de”]; [“fghi”] with “j” gives [“abc”]; [“de”]; [“fgji”]

(f) #[nat]
§ 1 because “A nonempty bunch of items is also an item.” page 17
or, informally
   #[nat]
   = #[0, 1, 2, 3, ...]
   = #([0], [1], [2], [3], ...)
   = #([0], #[1], #[2], #[3], ...)
   = 1, 1, 1, 1, ...
This is the sort of “proof” that mathematicians accept, but it’s not a formal proof because the three dots mean “guess what goes here”. Anyway, the question did not ask for proof.

(g) \[3; 3; \ldots\] 
\[\equiv\] 
\[\{\text{nil}, 3, 3;3, 3;3;3, \ldots\}\] 
\[\equiv\] 
\[\{\text{nil}, [3], [3;3], [3;3;3], \ldots\}\] 
\[\equiv\] 
\[\{\text{nil}, #[3], #[3;3], #[3;3;3], \ldots\}\] 
\[\equiv\] 
\[0, 1, 2, 3, \ldots\] 
\[\equiv\] 
\[\text{nat}\]
Again, an informal “proof”, but the question did not ask for proof.

(h) \[3; 4\] 
\[\equiv\] 
\[\{3; 4\}\] 
\[\equiv\] 
\[\{3; 4; \ldots\}\] 
\[\equiv\] 
\[\{3; 4; 12\}\] 
\[\equiv\] 
\[\{3; 4; 12\}\] 
which is all lists of 12 integers, and \[3; 4\] is not a list of 12 integers, so the answer is \(\bot\)

(i) \[3; 4\] 
\[\equiv\] 
\[\{3; 4\}\] 
\[\equiv\] 
\[\{3; 4; \ldots\}\] 
\[\equiv\] 
\[\{3; 4; 2\}\] 
\[\equiv\] 
\[\{3; 4; 2\}\] 
The list \[3; 4; \ldots\] is all lists of length 2 whose item 0 is 3 and whose item 1 is in \(\text{int}\). The list \[3; 4\] is one of them, so the answer is \(\top\)

(j) \[3, 4; 5\] 
\[\equiv\] 
\[\{3, 4; 5\}\] 
\[\equiv\] 
\[\{3, 4; 5\}\] 
\[\equiv\] 
\[\{3, 4; 5\}\] 
\[\equiv\] 
\[\{3, 4; 5\}\] 
and \[3\] is not a list of length 2, so the answer is \(\bot\)

(k) \[(3, 4); 5\] 
\[\equiv\] 
\[\{(3, 4); 5\}\] 
\[\equiv\] 
\[\{(3, 4); 5\}\] 
\[\equiv\] 
\[\{(3, 4); 5\}\] 
and both these lists are of length 2 and both items of each are in \(\text{int}\) so the answer is \(\top\)

(l) \[3; (4; 5); 6; (7, 8, 9)\] 
\[\equiv\] 
\[\{3; (4; 5); 6; (7, 8, 9)\}\] 
\[\equiv\] 
\[\{3; (4; 5); 6; (7, 8, 9)\}\] 
\[\equiv\] 
\[\{3; (4; 5); 6; (7, 8, 9)\}\] 
\[\equiv\] 
\[\{3; 4; 6; 7, 3; 4; 6; 7\}\] 
\[\equiv\] 
\[\{3; 4; 6; (7, 8)\}\]