60 Suppose we add laws to allow various operators to distribute over string join (semicolon). For example, if i and j are items and s and t are strings, then the laws

$$nil + nil \;=\; nil$$

$$(i;\, s) + (j;\, t) \;=\; i{+}j;\, s{+}t$$

say that strings are added item by item (a sum of strings is a string of sums). For example,

$$(2;\, 4;\, 7) + (3;\, 9;\, 1) \;=\; 5;\, 13;\, 8$$

What string f is defined by

$$f \;=\; 0;\, 1;\, f + f_{1;..\infty}$$

After trying the question, scroll down to the solution.

§ Using ... for unknown items, here are f and $f_{1;..\infty}$ and below that their sum.

$f = 0; 1; ...$
$f_{1;..\infty} = 1; ...$
$f + f_{1;..\infty} = 1; ...$

So now we know item 2. So again

$f = 0; 1; 1; ...$
$f_{1;..\infty} = 1; 1; ...$
$f + f_{1;..\infty} = 1; 2; ...$

So now we know item 3. So again

$f = 0; 1; 1; 2; ...$
$f_{1;..\infty} = 1; 1; 2; ...$
$f + f_{1;..\infty} = 1; 2; 3; ...$

So now we know item 4. And so on. It's the Fibonacci sequence

$f = 0; 1; 1; 2; 3; 5; 8; 13; 21; 34; ...$

usually defined as

$f_0 = 0$
$f_1 = 1$
$f_{n+2} = f_n + f_{n+1}$