

59 In Section 2.2 there is a self-describing expression

““““0;0;(0;..29);28;28;(1;..28)””””0;0;(0;..29);28;28;(1;..28)””””

- which evaluates to its own representation.
- (a) Write an expression that evaluates to twice its own representation. In other words, it evaluates to its own representation followed by its own representation again.
 - (b) Make it into a self-printing program. Let's say that `!e` prints the value of expression `e`.

After trying the question, scroll down to the solution.

Before answering the question, here is a guide to the self-reproducing expression

““““0;0;(0;..29);28;28;(1;..28)””””0;0;(0;..29);28;28;(1;..28)””””

To begin, we have an opening quotation mark “ . It starts a string of characters (a text). The next two opening quotation marks ““ are the way you write one opening quotation mark within a character string, as it says a couple of sentences earlier:

(but a double-quote character within the text must be written twice)

So the first character, character 0, within the string is an opening quotation mark “ . The second character, character 1, is a subscript zero character $_0$. The next character, character 2, is a subscript semi-colon ; . And so on. Character 27 is a subscript right parenthesis $)$. Next we have two closing quotation marks ”” ; that is the way you write one closing quotation mark within a character string, and it is character 28. Then there is one more closing quotation mark ” to end the character string. Let's call that character string s .

$s = \text{““““}0;0;(0;..29);28;28;(1;..28)\text{””””}$

After that we have another string of characters; let's call it i .

$i = 0;0;(0;..29);28;28;(1;..28)$

String i is a subscript, so it is indexing string s .

s_i

The first item in string i is 0 , so that's s_0 , which is an opening quotation mark “ .

The next item in string i is again 0 , so that's s_0 , which is again “ .

Next in string i we have items 0;..29 , so that indexes all of s , from its first character at index 0 , which is “ , to and including its last character at index 28 , which is ” . Remember that 0;..29 includes 0 but not 29 .

Next in string i we have 28 , so that's s_{28} , which is ” .

Next in string i we have another 28 , so that's s_{28} again, which is ” .

And finally in string i we have items 1;..28 , so that indexes all of s except for its first and last characters, which are the opening and closing quotation marks “ and ” .

If you have been keeping track of the characters of s indexed by i , they are:

““““0;0;(0;..29);28;28;(1;..28)””””0;0;(0;..29);28;28;(1;..28)””””

which is the self-describing expression, also known as a self-reproducing automaton.

- (a) Write an expression that evaluates to twice its own representation. In other words, it evaluates to its own representation followed by its own representation again.

§ ““““2*(0;0;(0;..33);32;32;(1;..32))””””2*(0;0;(0;..33);32;32;(1;..32))””””

- (b) Make it into a self-printing program. Let's say that $!e$ prints the value of expression e .

§ !“!“““0;1;0;1;(1;..34);33;33;(2;..33)””””0;1;0;1;(1;..34);33;33;(2;..33)””””

Here is a program that prints itself twice with a period between (for sequential composition).

!“!“““0;1;0;1;(1;..76);75;76;75;(2;..75);76;0;1;0;1;(1;..76);75;76;75;(2;..75)””””.””””
0;1;0;1;(1;..76);75;76;75;(2;..75);76;0;1;0;1;(1;..76);75;76;75;(2;..75)

When this program is executed, it prints a program that's twice as long. And when that program is executed, it prints a program that's four times as long as the original. And so on, with exponentially increasing length.