

57 The strings defined in Section 2.2 have natural indexes and extended natural lengths. Add a new operator, the inverse of join, to obtain strings that have negative indexes and lengths.

After trying the question, scroll down to the solution.

§ Let $\dot{-}$ be both a one-operand prefix operator with precedence level 2, and a two-operand infix operator with precedence level 4.

One idea is that subtracting a string wipes out the end, like this:

$$\text{"abcde"} \dot{-} \text{"de"} = \text{"abc"}$$

Here are some axioms to accomplish this. Let s , t , and u be finite strings.

$$\dot{-}s = \text{nil} \dot{-}s$$

$$\dot{-} \dot{-}s = s$$

$$\dot{-}(s; t) = \dot{-}t; \dot{-}s$$

$$s \dot{-}t = s; \dot{-}t$$

$$(s; t) \dot{-}t = s$$

$$s; (t \dot{-}u) = (s; t) \dot{-}u$$

$$s \dot{-}t = s \dot{-}u \iff t=u$$

$$s \dot{-}s = \text{nil}$$

$$\leftrightarrow \dot{-}s = -\leftrightarrow s$$

$$\leftrightarrow(s \dot{-}t) = \leftrightarrow s -\leftrightarrow t$$

If strings can also have infinite length, some of these axioms need antecedents saying that the lengths of some operands must be finite. Now we need to say something about indexing these negative strings. Maybe

$$(\dot{-}s)_n = s_{-n}$$

Another idea is that subtracting a string puts it backwards in front, like this:

$$\text{"abc"} \dot{-} \text{"de"} = \text{"edabc"}$$

except that the index of "e" is -2 , the index of "d" is -1 , the index of "a" is 0 , and so on up.