Process $P$ is an endless repetition of a “non-critical section” $PN$ and a “critical section” $PC$. Process $Q$ is similar.

\[
P = PN. PC. P
\]
\[
Q = QN. QC. Q
\]

They are executed concurrently $(P \parallel Q)$. Specify formally that the two critical sections are never executed at the same time.

(a) by inserting variables that are assigned but never used.

§ I introduce binary interactive variables $a$ and $b$ as follows:

\[
P = PN. a:= \top. PC. a:= \bot. P
\]
\[
Q = QN. b:= \top. QC. b:= \bot. Q
\]

Now mutual exclusion is specified as follows.

\[
\neg \exists t: t, .. \infty \cdot a t \land b t
\]

(b) by inserting outputs on channels that are never read.

§ I introduce binary channels $a$ and $b$ as follows:

\[
P = PN. a! \top. PC. a! \bot. P
\]
\[
Q = QN. b! \top. QC. b! \bot. Q
\]

Now mutual exclusion is specified as follows.

\[
\neg \exists i: wa, .. \infty \cdot \exists j: wb, .. \infty \cdot \mathcal{M}_i \land \mathcal{I}_a \leq \mathcal{I}_b < \mathcal{I}_a + 1 \lor \mathcal{M}_j \land \mathcal{I}_b \leq \mathcal{I}_a < \mathcal{I}_b + 1
\]

If we assume $PC$ and $QC$ take nonzero time we can specify mutual exclusion as follows:

\[
\neg \exists i: wa, .. \infty \cdot \exists j: wb, .. \infty \cdot \mathcal{M}_i \land \mathcal{M}_j \land (\mathcal{I}_a \leq \mathcal{I}_b < \mathcal{I}_a + 1 \lor \mathcal{I}_b \leq \mathcal{I}_a < \mathcal{I}_b + 1)
\]