

530 (mutual exclusion) Process  $P$  is an endless repetition of a “non-critical section”  $PN$  and a “critical section”  $PC$ . Process  $Q$  is similar.

$$P = PN. PC. P$$

$$Q = QN. QC. Q$$

They are executed concurrently  $(P \parallel Q)$ . Specify formally that the two critical sections are never executed at the same time

- (a) by inserting variables that are assigned but never used.
- (b) by inserting outputs on channels that are never read.

After trying the question, scroll down to the solution.

(a) by inserting variables that are assigned but never used.

§ I introduce binary interactive variables  $a$  and  $b$  as follows:

$$P = PN. a := \top. PC. a := \perp. P$$

$$Q = QN. b := \top. QC. b := \perp. Q$$

Now mutual exclusion is specified as follows.

$$\neg \exists t'': t, \dots \infty. a t' \wedge b t''$$

(b) by inserting outputs on channels that are never read.

§ I introduce binary channels  $a$  and  $b$  as follows:

$$P = PN. a! \top. PC. a! \perp. P$$

$$Q = QN. b! \top. QC. b! \perp. Q$$

Now mutual exclusion is specified as follows.

$$\neg \exists i: wa, \dots \infty. \exists j: wb, \dots \infty. \mathcal{M}a_i \wedge \mathcal{I}a_i \leq \mathcal{I}b_j < \mathcal{I}a_{i+1} \vee \mathcal{M}b_j \wedge \mathcal{I}b_j \leq \mathcal{I}a_i < \mathcal{I}b_{j+1}$$

If we assume  $PC$  and  $QC$  take nonzero time we can specify mutual exclusion as follows:

$$\neg \exists i: wa, \dots \infty. \exists j: wb, \dots \infty. \mathcal{M}a_i \wedge \mathcal{M}b_j \wedge (\mathcal{I}a_i \leq \mathcal{I}b_j < \mathcal{I}a_{i+1} \vee \mathcal{I}b_j \leq \mathcal{I}a_i < \mathcal{I}b_{j+1})$$