528√ Prove that execution of the following program deadlocks.

(a) $\textbf{new } c?\ int\cdot\ \ c?.\ \ c!\ 5$

(b) $\textbf{new } c, d?\ int\cdot\ \ (c?.\ \ d!\ 6) \parallel (d?.\ \ c!\ 7)$

After trying the question, scroll down to the solution.

(a) $\textbf{new}\ c?\ int\cdot\ \ c?.\ \ c!\ 5$

§ Inserting the wait for input,

$$\textbf{new}\ c?\ int\cdot\ \ t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ c?.\ \ c!\ 5$$

$$=\ \ \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \exists\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{w}, \boldsymbol{w}'{:}\ xnat\cdot$$
$$\boldsymbol{r}{=}\boldsymbol{w}{=}0\ \land\ (t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ \boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.\ \ \mathcal{M}_{\boldsymbol{w}}=5\ \land\ \mathcal{T}_{\boldsymbol{w}}=t\ \land(\boldsymbol{w}{:=}\ \boldsymbol{w}{+}1))$$

First, expand $\boldsymbol{w}{:=}\ \boldsymbol{w}{+}1$, taking $\boldsymbol{r}$, $\boldsymbol{w}$, x, and t as the state variables.

$$=\ \ \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \exists\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{w}, \boldsymbol{w}'{:}\ xnat\cdot$$
$$\boldsymbol{r}{=}\boldsymbol{w}{=}0$$
$$\land\ (t{:=}\ t\uparrow(\mathcal{T}_{\boldsymbol{r}}+1).\ \ \boldsymbol{r}{:=}\ \boldsymbol{r}{+}1.$$
$$\mathcal{M}_{\boldsymbol{w}}=5\ \land\ \mathcal{T}_{\boldsymbol{w}}=t\ \land\ \boldsymbol{r}'{=}\boldsymbol{r}\ \land\ \boldsymbol{w}'=\boldsymbol{w}{+}1\ \land\ x'{=}x\ \land\ t'{=}t)$$

Now use the Substitution Law twice and one-point twice.

$$=\ \ \exists\mathcal{M}{:}\ \infty{*}int\cdot\ \exists\mathcal{T}{:}\ \infty{*}xnat\cdot\ \exists\boldsymbol{r}', \boldsymbol{w}'{:}\ xnat\cdot$$
$$\mathcal{M}_0=5\ \land\ \mathcal{T}_0=t\uparrow(\mathcal{T}_0+1)\ \land\ \boldsymbol{r}'{=}1\ \land\ \boldsymbol{w}'{=}1\land x'{=}x\ \land t'=t\uparrow(\mathcal{T}_0+1)$$

Look at the conjunct $\mathcal{T}_0=t\uparrow(\mathcal{T}_0+1)$. It says $\mathcal{T}_0=\infty$.

$$=\ \ x'{=}x\ \land\ t'{=}\infty$$

The theory tells us that execution takes forever because the wait for input is infinite.

(b) $\textbf{new}\ c, d?\ int\cdot\ \ (c?.\ \ d!\ 6)\ \|\ (d?.\ \ c!\ 7)$

§ Inserting the input waits, we get

$$\textbf{new}\ c, d?\ int\cdot\ \ (t{:=}\ t\uparrow(\mathcal{T}c_{\boldsymbol{rc}}+1).\ \ c?.\ \ d!\ 6)\ \|\ (t{:=}\ t\uparrow(\mathcal{T}d_{\boldsymbol{rd}}+1).\ \ d?.\ c!7)$$

after a little work, we obtain

$$=\ \ \exists\mathcal{M}c, \mathcal{M}d{:}\ \infty{*}int\cdot\ \exists\mathcal{T}c, \mathcal{T}d{:}\ \infty{*}xnat\cdot\ \exists\boldsymbol{rc}, \boldsymbol{rc}', \boldsymbol{wc}, \boldsymbol{wc}', \boldsymbol{rd}, \boldsymbol{rd}', \boldsymbol{wd}, \boldsymbol{wd}'{:}\ xnat\cdot$$
$$\mathcal{M}d_0=6\ \land\ \mathcal{T}d_0=t\uparrow(\mathcal{T}c_0+1)\ \land\ \mathcal{M}c_0=7\ \land\ \mathcal{T}c_0=t\uparrow(\mathcal{T}d_0+1)$$
$$\land\ \ \boldsymbol{rc}'{=}\boldsymbol{wc}'{=}\boldsymbol{rd}'{=}\boldsymbol{wd}'{=}1\land x'{=}x\ \land\ t'=t\uparrow(\mathcal{T}c_0+1)\uparrow t\uparrow(\mathcal{T}d_0+1)$$

The conjuncts $\mathcal{T}d_0=t\uparrow(\mathcal{T}c_0+1)$ and $\mathcal{T}c_0=t\uparrow(\mathcal{T}d_0+1)$ tell us that $\mathcal{T}d_0=\mathcal{T}c_0=\infty$.

$$=\ \ x'{=}x\ \land\ t'{=}\infty$$