§ Let $p$ be the door where the prize is. That's
\[(p' = 0)/2 + (p' = 1)/3 + (p' = 2)/6\]
Suppose the contestant chooses door 0 and Monty opens door 1, which tells the contestant that the prize is not behind door 1.
\[p' = 1\]
We know two distributions, so we multiply them together. But multiplying distributions does not necessarily result in a distribution, so we must normalize the result to obtain a distribution.
\[\frac{((p' = 0)/2 + (p' = 1)/3 + (p' = 2)/6) \times (p' = 1)}{((\Sigma p' \cdot (p' = 0)/2 + (p' = 1)/3 + (p' = 2)/6) \times (p' = 1))}\]
\[= \frac{(p' = 0) \times (p' = 1)/2 + (p' = 1) \times (p' = 1)/3 + (p' = 2) \times (p' = 1)/6}{(1/2 + 0 + 1/6)}\]
\[= \frac{(p' = 0)/2 + (p' = 2)/6}{(2/3)}\]
\[= \frac{(p' = 0) \times 3/4 + (p' = 2) \times 1/4}{(2/3)}\]
So the prize is more probably behind door 0, and the contestant should stick with their choice of door 0.

Suppose the contestant chooses door 0 and Monty opens door 2, which tells the contestant that the prize is not behind door 2.
\[p' = 2\]
We know two distributions, so we multiply them together and normalize.
\[\frac{((p' = 0)/2 + (p' = 1)/3 + (p' = 2)/6) \times (p' = 2)}{((\Sigma p' \cdot (p' = 0)/2 + (p' = 1)/3 + (p' = 2)/6) \times (p' = 2))}\]
\[= \frac{(p' = 0) \times (p' = 2)/2 + (p' = 1) \times (p' = 2)/3 + (p' = 2) \times (p' = 2)/6}{(1/2 + 1/3 + 0)}\]
\[= \frac{(p' = 0)/2 + (p' = 1)/3}{(5/6)}\]
\[= \frac{(p' = 0) \times 3/5 + (p' = 1) \times 2/5}{(5/6)}\]
So the prize is again more probably behind door 0, and the contestant should stick with their choice of door 0.

Suppose the contestant chooses door 1 and Monty opens door 0. The resulting distribution is
\[\frac{((p' = 0)/2 + (p' = 1)/3 + (p' = 2)/6) \times (p' = 0)}{((\Sigma p' \cdot (p' = 0)/2 + (p' = 1)/3 + (p' = 2)/6) \times (p' = 0))}\]
\[= \frac{(p' = 0) \times (p' = 0)/2 + (p' = 1) \times (p' = 0)/3 + (p' = 2) \times (p' = 0)/6}{(0 + 1/3 + 1/6)}\]
\[= \frac{(p' = 1)/3 + (p' = 2)/6}{(1/2)}\]
\[= \frac{(p' = 1) \times 2/3 + (p' = 2) \times 1/3}{(1/2)}\]
So the prize is more probably behind door 1, and the contestant should stick.

Suppose the contestant chooses door 1 and Monty opens door 2. The resulting distribution is
\[\frac{((p' = 0)/2 + (p' = 1)/3 + (p' = 2)/6) \times (p' = 2)}{((\Sigma p' \cdot (p' = 0)/2 + (p' = 1)/3 + (p' = 2)/6) \times (p' = 2))}\]
same as before.
\[
\frac{1}{2} \left( (p' = 0) \times 3/5 \, + \, (p' = 1) \times 2/5 \right)
\]

So the prize is more probably behind door 0, and the contestant should switch.

Suppose the contestant chooses door 2 and Monty opens door 0. The resulting distribution is

\[
(p' = 1) \times 2/3 \, + \, (p' = 2) \times 1/3
\]

So the prize is more probably behind door 1, and the contestant should switch.

Suppose the contestant chooses door 2 and Monty opens door 1. The resulting distribution is

\[
(p' = 0) \times 3/4 \, + \, (p' = 2) \times 1/4
\]

So the prize is more probably behind door 0, and the contestant should switch.

In addition to finding out whether to stick or switch, we also find out that the contestant maximizes their chance of winning \( ((2/3 + 3/4)/2 = 17/24) \) by choosing door 2 and switching.