5.1 The notation \( \text{do } P \text{ while } b \text{ od} \) has been used as a loop construct that is executed as follows. First, \( P \) is executed; then \( b \) is evaluated, and if its value is \( \top \) then execution is repeated, and if its value is \( \bot \) then execution is finished. Let the program variable be integer variable \( x \). Prove that the specification 
\[
\text{mod } x' \ 2 = \text{mod } x \ 2
\]
is refined by the loop 
\[
\text{do } x := x - 2 \text{ while } x \geq 2 \text{ od}
\]

§ To prove \( S \) is refined by \( \text{do } P \text{ while } b \text{ od} \), prove instead 
\[
S \iff P \text{ if } b \text{ then } S \text{ else } \text{ok fi}
\]
So we prove 
\[
(mod \ x' \ 2 = \text{mod } x \ 2 \iff x := x - 2. \text{ if } x \geq 2 \text{ then } mod \ x' \ 2 = \text{mod } x \ 2 \text{ else } \text{ok fi})
\]
replace \( \text{ok} \)
\[
= (mod \ x' \ 2 = \text{mod } x \ 2 \iff x := x - 2. \text{ if } x \geq 2 \text{ then } mod \ x' \ 2 = \text{mod } x \ 2 \text{ else } x' = x \ 2 \text{ fi})
\]
substitution
\[
= mod \ x' \ 2 = \text{mod } x \ 2 \iff \text{if } x - 2 \geq 2 \text{ then } mod \ x' \ 2 = \text{mod } (x - 2) \ 2 \text{ else } x' = x - 2 \text{ fi}
\]
by cases
\[
= (mod \ x' \ 2 = \text{mod } x \ 2 \iff x - 2 \geq 2 \land mod \ x' \ 2 = \text{mod } (x - 2) \ 2) \text{ specialization and}
\]
\[
\land (mod \ x' \ 2 = \text{mod } x \ 2 \iff x - 2 < 2 \land x' = x - 2) \text{ specialization again}
\]
\[
\iff (mod \ x' \ 2 = \text{mod } x \ 2 \iff mod \ x' \ 2 = \text{mod } (x - 2) \ 2) \text{ context and}
\]
\[
\land (mod \ x' \ 2 = \text{mod } x \ 2 \iff x' = x - 2) \text{ context again}
\]
\[
= (mod \ (x - 2) \ 2 = \text{mod } x \ 2 \iff mod \ x' \ 2 = \text{mod } (x - 2) \ 2)
\]
\[
\land (mod \ (x - 2) \ 2 = \text{mod } x \ 2 \iff x' = x - 2)
\]
\[
= \top \land \top
\]
\[
= \top
\]