498 Let a and b be binary interactive variables. Define

$$loop = \textbf{if } b \textbf{ then } loop \textbf{ else } ok \textbf{ fi}$$

Add a time variable according to any reasonable measure, and then express

$$b{:=}\bot \;\|\; loop$$

as an equivalent program but without using $\|$.

After trying the question, scroll down to the solution.

§ The left process owns b . Variable a could belong to either process; let's give it to the left process also. Let assignment take time 1 .

$b{:=}\bot \;\|\; loop$ — definition of assignment and $loop$ with time

$= \quad b\,t' = \bot \;\land\; a\,t' = a\,t \;\land\; t' = t{+}1 \;\|\; \textbf{if } b\,t \textbf{ then } t{:=}\,t{+}1.\; loop \textbf{ else } ok \textbf{ fi}$

on the left, use context; on the right, use definition of $loop$ again and ok

$= \quad b(t{+}1){=}\bot \;\land\; a(t{+}1) = a\,t \;\land\; t' = t{+}1$
$\quad \| \; \textbf{if } b\,t \textbf{ then } t{:=}\,t{+}1.\; \textbf{if } b\,t \textbf{ then } t{:=}\,t{+}1.\; loop \textbf{ else } t'{=}t \textbf{ fi else } t'{=}t \textbf{ fi}$

substitution law

$= \quad b(t{+}1){=}\bot \;\land\; a(t{+}1) = a\,t \;\land\; t' = t{+}1$
$\quad \| \; \textbf{if } b\,t \textbf{ then if } b(t{+}1) \textbf{ then } t{:=}\,t{+}2.\; loop \textbf{ else } t'{=}t{+}1 \textbf{ fi else } t'{=}t \textbf{ fi}$

definition of $\|$

$= \quad \exists tL, tR\cdot \; b(t{+}1){=}\bot \;\land\; a(t{+}1) = a\,t \;\land\; tL = t{+}1$
$\quad \land\; \textbf{if } b\,t \textbf{ then if } b(t{+}1) \textbf{ then } tR = t{+}2.\; loop \textbf{ else } tR = t{+}1 \textbf{ fi else } tR{=}t \textbf{ fi}$
$\quad \land\; t' = tL{\uparrow}tR$

context: $b(t{+}1){=}\bot$

$= \quad \exists tL, tR\cdot \; b(t{+}1){=}\bot \;\land\; a(t{+}1) = a\,t \;\land\; tL = t{+}1$
$\quad \land\; \textbf{if } b\,t \textbf{ then if } \bot \textbf{ then } tR = t{+}2.\; loop \textbf{ else } tR = t{+}1 \textbf{ fi else } tR{=}t \textbf{ fi}$
$\quad \land\; t' = tL{\uparrow}tR$

case base

$= \quad \exists tL, tR\cdot \; b(t{+}1){=}\bot \;\land\; a(t{+}1) = a\,t \;\land\; tL = t{+}1$
$\quad \land\; \textbf{if } b\,t \textbf{ then } tR = t{+}1 \textbf{ else } tR{=}t \textbf{ fi}$ — factor out $tR{=}$
$\quad \land\; t' = tL{\uparrow}tR$

$= \quad \exists tL, tR\cdot \; b(t{+}1){=}\bot \;\land\; a(t{+}1) = a\,t \;\land\; tL = t{+}1$
$\quad \land\; tR = \textbf{if } b\,t \textbf{ then } t{+}1 \textbf{ else } t \textbf{ fi}$
$\quad \land\; t' = tL{\uparrow}tR$ — one-point for both tL and tR

$= \quad b(t{+}1){=}\bot \;\land\; a(t{+}1) = a\,t \;\land\; t' = (t{+}1){\uparrow}\textbf{if } b\,t \textbf{ then } t{+}1 \textbf{ else } t \textbf{ fi}$ — factor in $(t{+}1){\uparrow}$

$= \quad b(t{+}1){=}\bot \;\land\; a(t{+}1) = a\,t \;\land\; t' = \textbf{if } b\,t \textbf{ then } (t{+}1){\uparrow}(t{+}1) \textbf{ else } (t{+}1){\uparrow}t \textbf{ fi}$

simplify the two $\uparrow$

$= \quad b(t{+}1){=}\bot \;\land\; a(t{+}1) = a\,t \;\land\; t' = \textbf{if } b\,t \textbf{ then } t{+}1 \textbf{ else } t{+}1 \textbf{ fi}$ — case idempotent

$= \quad b(t{+}1){=}\bot \;\land\; a(t{+}1) = a\,t \;\land\; t' = t{+}1$ — context

$= \quad b\,t' = \bot \;\land\; a\,t' = a\,t \;\land\; t' = t{+}1$

$= \quad b{:=}\bot$