Exercise 161 asks for a program to compute cumulative sums (running total). Write a program that can be transformed from sequential to concurrent execution with $\log n$ time where $n$ is the length of the list.

Let $L$ be a list variable. The result we want is $R \, 0 \, (\#L)$ where

$$R \, i \, k \equiv \#L' = \#L \land \forall j: i \ldots k \cdot L' \, j = \sum L \, i \ldots j + 1$$

We refine it as follows.

$$R \, i \, k \Leftarrow \begin{cases} \text{if } k - i \leq 1 \text{ then } & \text{ok} \\ \text{else} & \text{var } m := \text{div} \, (k + i) / 2. \\ & (R \, i \, m \parallel R \, m \, k). \\ & \text{for } j := m \ldots k \text{ do } L \, j := L \, j \, + \, L \, (m - 1) \text{ od} \text{ fi} \end{cases}$$

The final for-loop can be concurrent, so that it takes no time. The computation then takes $\log (\#L)$ time.