489 We want to find the smallest number in $0,..n$ with property p. Linear search solves the problem. But evaluating p is expensive; let us say it takes time 1, and all else is free. The fastest solution is to evaluate p on all n numbers concurrently, and then find the smallest number that has the property. Write a program without concurrency for which the sequential to concurrent transformation gives the desired computation.

After trying the question, scroll down to the solution.

§ We introduce array A: $[n*bin]$. We define the desired result R , invariant $I\ i$, and helper specification P as follows.

$$R \;=\; \neg(\exists j{:}\ 0,..h'\cdot\ p\ j) \wedge (p\ h' \vee h'=n)$$
$$I\ i \;=\; \forall j{:}\ 0,..i\cdot\ A\ j = p\ j$$
$$P \;=\; I\ n \wedge \neg(\exists j{:}\ 0,..h\cdot\ p\ j) \;\Rightarrow\; R$$

Now the program is

$$R \;\Leftarrow\; I\,0{\Rightarrow}I'n.\ \ h{:}= 0.\ \ P$$
$$I\,0 \Rightarrow I'n \;\Leftarrow\; \mathbf{for}\ i{:}= 0;..n\ \mathbf{do}\ I\ i \Rightarrow I'(i+1)\ \mathbf{od}$$
$$I\ i \Rightarrow I'(i+1) \;\Leftarrow\; A\ i{:}= p\ i$$
$$P \;\Leftarrow\; \mathbf{if}\ h{=}n\ \mathbf{then}\ ok\ \mathbf{else}\ \mathbf{if}\ A\ h\ \mathbf{then}\ ok\ \mathbf{else}\ h{:}= h+1.\ \ P\ \mathbf{fi}\ \mathbf{fi}$$

The n iterations of the **for**-loop can be executed concurrently.

We can express the result of the sequential to concurrent transformation at source as follows.

$$R \;\Leftarrow\; I\,0 \Rightarrow I'n.\ \ h{:}= 0.\ \ P$$
$$I\,0 \Rightarrow I'n \;\Leftarrow\; i{:}= 0.\ \ I\ i \Rightarrow I'n$$
$$I\ i \Rightarrow I'n \;\Leftarrow\; \mathbf{if}\ i{=}n\ \mathbf{then}\ ok\ \mathbf{else}\ A\ i{:}= p\ i\ \|\ (i{:}= i+1.\ \ I\ i \Rightarrow I'n)\ \mathbf{fi}$$
$$P \;\Leftarrow\; \mathbf{if}\ h{=}n\ \mathbf{then}\ ok\ \mathbf{else}\ \mathbf{if}\ A\ h\ \mathbf{then}\ ok\ \mathbf{else}\ h{:}= h+1.\ \ P\ \mathbf{fi}\ \mathbf{fi}$$

To understand the execution, it might help to unroll the recursion a little: in the refinement of $I\ i \Rightarrow I'n$, replace the recursive call $I\ i \Rightarrow I'n$ by what it calls:

$$\mathbf{if}\ i{=}n\ \mathbf{then}\ ok\ \mathbf{else}\ A\ i{:}= p\ i\ \|\ (i{:}= i+1.\ \ I\ i \Rightarrow I'n)\ \mathbf{fi}$$

And maybe do the same once more.