(disjoint composition) Concurrent composition $P \parallel Q$ requires that $P$ and $Q$ have no variables in common, although each can make use of the initial values of the other's variables by making a private copy. An alternative, let's say disjoint composition, is to allow both $P$ and $Q$ to use all the variables with no restrictions, and then to choose disjoint sets of variables $v$ and $w$ and define

$$P \mid v \parallel w \mid Q \equiv (P. \; v'=v) \land (Q. \; w'=w)$$

(a) Describe how $P \mid v \parallel w \mid Q$ can be executed.

§ Make a copy of all variables. Execute $P$ using the original set of variables and concurrently execute $Q$ using the copies. Then copy back from the copy $w$ to the original $w$. Then throw away the copies. There may be variables $x$ other than $v$ and $w$; if so, their final values are arbitrary, and this implementation makes them be what $P$ says they should be. Formally, using application $\langle v. \; P \rangle \; x$ as the formal notation for (substitute $x$ for $v$ in $P$),

$$\text{var } cv := v. \; \text{var } cw := w. \; \text{var } cx := x.$$  

$$(P \parallel \langle v, w, x, v', w', x' \parallel Q \rangle) \; cv \; cw \; cx \; cv \;' \; cw \; cx \'. \; w := cw$$

(b) Prove that if $P$ and $Q$ are implementable specifications, then $P \mid v \parallel w \mid Q$ is implementable.

§ First, a lemma.

$$P, \; v'=v \quad \text{expand sequential composition}$$

$$\equiv \exists v'', w'', x'', (v', w', x' \parallel P) \; v'' \; w'' \; x'' \land v'=v'' \quad \text{one-point } v''$$

$$\equiv \exists w'', x'', (v', w', x' \parallel P) \; v' \; w'' \; x'' \quad \text{rename } w'', x'' \text{ to } w', x'$$

$$\equiv \exists w', x' (v', w', x' \parallel P) \; v' \; w' \; x' \quad \text{simplify}$$

$$\equiv \exists w', x' \; P.$$

So

$$P \mid v \parallel w \mid Q \equiv (P. \; v'=v) \land (Q. \; w'=w) \equiv (\exists w', x' \parallel P) \land (\exists v', x' \parallel Q)$$

Now the main proof.

(S $P \mid v \parallel w \mid Q$ is implementable) \hspace{2cm} \text{definition of implementable}

$$\equiv \forall v, w, x. \exists v', w', x' \; P \mid v \parallel w \mid Q \quad \text{use previous result}$$

$$\equiv \forall v, w, x. \exists v'. \; (\exists w', x' \parallel P) \land (\exists v', x' \parallel Q) \quad \text{idem for } x'$$

$$\equiv \forall v, w, x. \exists v', (\exists w', x' \parallel P) \land (\exists v', x' \parallel Q) \quad \text{distribution (factoring)}$$

$$\equiv \forall v, w, x. \exists v', (\exists w', x' \parallel P) \land (\exists v', x' \parallel Q) \quad \text{distribution (factoring)}$$

$$\equiv \forall v, w, x. (\exists v', \exists w', x' \parallel P) \land (\exists v', x' \parallel Q) \quad \text{splitting law}$$

$$\equiv (\forall v, w, x. \exists v', w', x' \parallel P) \land (\forall v, w, x. \exists v', x' \parallel Q) \quad \text{definition of implementable}$$

$$\equiv (P \; \text{is implementable}) \land (Q \; \text{is implementable})$$