First, a lemma.

(b) Prove that if

\[ P \mid \mid Q \]

have no variables in common, although each can make use of the initial values of the other's variables by making a private copy. An alternative, let's say disjoint composition, is to allow both \( P \) and \( Q \) to use all the variables with no restrictions, and then to choose disjoint sets of variables \( v \) and \( w \) and define

\[ P \mid \mid v \mid \mid w \mid Q = (P. \ v'=v) \land (Q. \ w'=w) \]

(a) Describe how \( P \mid \mid v \mid \mid w \mid Q \) can be executed.

Make a copy of all variables. Execute \( P \) using the original set of variables and concurrently execute \( Q \) using the copies. Then copy back from the copy \( w \) to the original \( w \). Then throw away the copies. There may be variables \( x \) other than \( v \) and \( w \); if so, their final values are arbitrary, and this implementation makes them be what \( P \) says they should be. Formally, using application \( x \to P \) as the formal notation for (substitute \( x \) for \( v \) in \( P \)),

\[
\text{var} \ cv := v \cdot \text{var} \ cw := w \cdot \text{var} \ cx := x \cdot \\
(P \mid \mid \langle v, w, x, v', w', x' \to Q \rangle) cv \ cw \ cx cv' \ cx'. w := cw
\]

(b) Prove that if \( P \) and \( Q \) are implementable specifications, then \( P \mid \mid v \mid \mid w \mid Q \) is implementable.

First, a lemma.

\[
\begin{align*}
P \cdot \cdot v' & = v \\
& \equiv \exists v'', w'', x'' \cdot (v', w', x' \to P) v'' w'' x'' \land v' = v'' & \text{expand sequential composition} \\
& \equiv \exists w'', x'' \cdot (v', w', x' \to P) v' w'' x'' & \text{rename } w'', x'' \text{ to } w', x' \\
& \equiv \exists w', x' \cdot (v', w', x' \to P) v' w' x' & \text{simplify} \\
& \equiv \exists w', x' \cdot P
\end{align*}
\]

So \( P \mid \mid v \mid \mid w \mid Q = (P. \ v' = v) \land (Q. \ w' = w) = (\exists w', x'. P) \land (\exists v', x'. Q) \)

Now the main proof.

\[
\begin{align*}
(P \mid \mid v \mid \mid w \mid Q \text{ is implementable}) & \equiv (P \mid \mid v \mid \mid w \mid Q \text{ is implementable}) \\
& \equiv (P \mid \mid v \mid \mid w \mid Q \text{ is implementable}) & \text{definition of implementable} \\
& \equiv (P \mid \mid v \mid \mid w \mid Q \text{ is implementable}) & \text{definition of implementable}
\end{align*}
\]

\[
\begin{align*}
& (P \mid \mid v \mid \mid w \mid Q \text{ is implementable}) \land (Q \mid \mid v \mid \mid w \mid Q \text{ is implementable}) \\
& \equiv (P \mid \mid v \mid \mid w \mid Q \text{ is implementable}) \land (Q \mid \mid v \mid \mid w \mid Q \text{ is implementable})
\end{align*}
\]