479 Let x and y be natural variables. Ignoring time, rewrite the following program as a program that does not use $\|$.

(a) $x := x+1 \;\|\; \textbf{if } x=0 \textbf{ then } y := 1 \textbf{ else } ok \textbf{ fi}$

(b) $\textbf{if } x>0 \textbf{ then } y := x-1 \textbf{ else } ok \textbf{ fi} \;\|\; \textbf{if } x=0 \textbf{ then } x := y+1 \textbf{ else } ok \textbf{ fi}$

After trying the question, scroll down to the solution.

(a)
$x := x+1 \parallel$ **if** $x=0$ **then** $y := 1$ **else** ok **fi**

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$x := x+1 \parallel$ **if** $x=0$ **then** $y := 1$ **else** ok **fi** — expand assignments and ok

$=$ $x' = x+1 \parallel$ **if** $x=0$ **then** $y'=1$ **else** $y'=y$ **fi** — concurrent composition

$=$ $x' = x+1 \wedge$ **if** $x=0$ **then** $y'=1$ **else** $y'=y$ **fi** — distribution

$=$ **if** $x=0$ **then** $x' = x+1 \wedge y'=1$ **else** $x' = x+1 \wedge y'=y$ **fi** — substitution law and identity

$=$ **if** $x=0$ **then** $y := 1.\ x' = x+1 \wedge y'=y$ **else** $ok.\ x' = x+1 \wedge y'=y$ **fi** — assignment

$=$ **if** $x=0$ **then** $y := 1.\ x := x+1$ **else** $ok.\ x := x+1$ **fi** — distribution

$=$ **if** $x=0$ **then** $y := 1$ **else** ok **fi**. $x := x+1$

(b)
if $x>0$ **then** $y := x-1$ **else** ok **fi** $\parallel$ **if** $x=0$ **then** $x := y+1$ **else** ok **fi**

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if $x>0$ **then** $y := x-1$ **else** ok **fi** $\parallel$ **if** $x=0$ **then** $x := y+1$ **else** ok **fi** — asmts and ok

$=$ **if** $x>0$ **then** $y' = x-1$ **else** $y'=y$ **fi** $\parallel$ **if** $x=0$ **then** $x' = y+1$ **else** $x'=x$ **fi** — conc. comp.

$=$ **if** $x>0$ **then** $y' = x-1$ **else** $y'=y$ **fi** $\wedge$ **if** $x=0$ **then** $x' = y+1$ **else** $x'=x$ **fi** — x is natural

$=$ **if** $x>0$ **then** $y' = x-1$ **else** $y'=y$ **fi** $\wedge$ **if** $\neg(x>0)$ **then** $x' = y+1$ **else** $x'=x$ **fi** case revers.

$=$ **if** $x>0$ **then** $y' = x-1$ **else** $y'=y$ **fi** $\wedge$ **if** $x>0$ **then** $x'=x$ **else** $x'=y+1$ **fi** case distributive

$=$ **if** $x>0$ **then** $y' = x-1 \wedge x'=x$ **else** $x'=y+1 \wedge y'=y$ **fi** — assignment twice

$=$ **if** $x>0$ **then** $y := x-1$ **else** $x := y+1$ **fi**