Implementer's variables $p, q$: real represent two points along a line. Each number tells the distance of one point from the origin (a standard point). They must be reimplemented by one implementer's variable $r$: real which tells the distance from $p$ to $q$. For examples, if $p=3$ and $q=5$, then $r=2$; if $p=5$ and $q=3$, then $r=-2$.

(a) What is the data transformer?

§ $r = q-p$

(b) A user has binary variable $b$ and operation

$compare \equiv b:=q \geq p$

Use your transformer from part (a) to transform operation $compare$.

§

\[
\forall p, q \cdot r = q-p \Rightarrow \exists p', q' \cdot r' = q'-p' \wedge (b:=q \geq p)
\]

expand assignment

\[
\equiv \forall p, q \cdot r = q-p \Rightarrow r' = q-p \wedge b' = q \geq p \wedge p'=p \wedge q'=q
\]

one-point twice

\[
\equiv \forall p, q \cdot r = q-p \Rightarrow r' = q-p \wedge b' = q \geq p
\]

context

\[
\equiv \forall p, q \cdot r = q-p \Rightarrow r'=r \wedge b'=r \geq 0
\]

some law of arithmetic

\[
\equiv \forall p, q \cdot p = q-r \Rightarrow r'=r \wedge b'=r \geq 0
\]

one-point and idempotent

\[
\equiv r'=r \wedge b'=r \geq 0
\]

definition of assignment

\[
\equiv b:=r \geq 0
\]