

471 The user's variable is natural n . The implementer's variable is set S . The operations are

$$\textit{start} = S := \{\textit{null}\}$$

$$\textit{insert} = S := S \cup \{S\}$$

$$\textit{ask} = n := |S|$$

Operation *start* starts variable S at the empty set. Then repeated use of operation *insert* increases the size of the set. Operation *ask* asks how large the set is. Reimplement this theory replacing the old implementer's variable S with natural variable m .

- (a) What is the data transformer?
- (b) Transform operation *ask*.

After trying the question, scroll down to the solution.

(a) What is the data transformer?

$$\S \quad m = \$S$$

(b) Transform operation ask .

$$\begin{aligned} \S & \quad \forall S. m = \$S \Rightarrow \exists S'. m' = \$S' \wedge ask && \text{replace } ask \\ = & \quad \forall S. m = \$S \Rightarrow \exists S'. m' = \$S' \wedge n' = \$S \wedge S'=S && \text{one-point } S' \\ = & \quad \forall S. m = \$S \Rightarrow m' = \$S \wedge n' = \$S && \text{context} \\ = & \quad \forall S. m = \$S \Rightarrow m'=m \wedge n'=m && \text{antidistributive} \\ = & \quad (\exists S. m = \$S) \Rightarrow m'=m \wedge n'=m \\ = & \quad \top \Rightarrow m'=m \wedge n'=m && \text{identity} \\ = & \quad m'=m \wedge n'=m && \text{assignment} \\ = & \quad n:=m \end{aligned}$$