468 (weak limited program bunches) Given natural number $n$, a theory maintains a subbunch of $0,..n$. The operations are: *mkempty*, which makes the bunch empty; *insert* $x$, which inserts $x$ into the bunch; *remove* $x$, which removes $x$ if it was there, and *check* $x$ which tells whether $x$ is there by assigning to a user's binary variable $u$.

(a) Design axioms that are weak enough to allow other operations to be added to the theory.

(b) Implement your theory of part (a) as a list of binary values.

(c) Transform your implementation of part (b) to one that maintains a list of natural numbers.

After trying the question, scroll down to the solution.

(a) Design axioms that are weak enough to allow other operations to be added to the theory.

§ The axioms use an auxiliary operation *preserve x* that does not affect whether *x* is in the bunch.

$$mkempty.\ preserve\ x.\ check\ x \Rightarrow \neg u'$$
$$insert\ x.\ preserve\ x.\ check\ x \Rightarrow u'$$
$$remove\ x.\ preserve\ x.\ check\ x \Rightarrow \neg u'$$
$$preserve\ x \Leftarrow ok$$
$$preserve\ x \Leftarrow check\ x$$
$$preserve\ x \Leftarrow insert\ y \wedge x \neq y$$
$$preserve\ x \Leftarrow remove\ y \wedge x \neq y$$
$$preserve\ x \Leftarrow preserve\ x.\ preserve\ x$$

(b) Implement your theory of part (a) as a list of binary values.

(c) Transform your implementation of part (b) to one that maintains a list of natural numbers.