468 (weak limited program bunches) Given natural number $n$, a theory maintains a subbunch of $0..n$. The operations are: *mkempty*, which makes the bunch empty; *insert $x$*, which inserts $x$ into the bunch; *remove $x$*, which removes $x$ if it was there, and

*check $x$* which tells whether $x$ is there by assigning to a user's binary variable $u$.

(a) Design axioms that are weak enough to allow other operations to be added to the theory. § The axioms use an auxiliary operation *preserve $x$* that does not affect whether $x$ is in the bunch.

- $mkempty. ~ preserve x. ~ check x => \neg u'$
- $insert x. ~ preserve x. ~ check x => u'$
- $remove x. ~ preserve x. ~ check x => \neg u'$
- $preserve x <= ok$
- $preserve x <= check x$
- $preserve x <= insert y \land x \neq y$
- $preserve x <= remove y \land x \neq y$
- $preserve x <= preserve x. ~ preserve x$

(b) Implement your theory of part (a) as a list of binary values.

(c) Transform your implementation of part (b) to one that maintains a list of natural numbers.