Let $p$ be a user's binary variable, and let $m$ be an implementer's natural variable. The operations allow the user to assign a value $n$ to the implementer's variable, and to test whether the implementer's variable is a prime number.

\[
\text{assign } n = m := n \\
\text{check } = p := \text{prime } m
\]

assuming \textit{prime} is suitably defined. If \textit{prime} is an expensive function, and the \textit{check} operation is more frequent than the \textit{assign} operation, we can improve the solution by making \textit{check} less expensive even if that makes \textit{assign} more expensive. Using data transformation, make this improvement.

After trying the question, scroll down to the solution.
§ I replace the implementer's natural variable $m$ by a new implementer's binary variable $q$. The data transformer is

$$ q = \text{prime } m $$

We have to check that this is a data transformer.

$$ \forall q \exists m : q = \text{prime } m $$

$\implies (\exists m : \top = \text{prime } m) \land (\exists m : \bot = \text{prime } m)$

generalization twice

$\iff (\top = \text{prime } 2) \land (\bot = \text{prime } 4)$

$\equiv \top$

Using this transformer, $\text{assign } n$ is transformed to

$$ \forall m : q = \text{prime } m \Rightarrow \exists m' : q' = \text{prime } m' \land (m := n) $$

expand assignment

$\equiv \forall m : q = \text{prime } m \Rightarrow \exists m' : q' = \text{prime } m' \land m' = n \land p' = p$

one-point $m'$

$\equiv \forall m : q = \text{prime } m \Rightarrow q' = \text{prime } n \land p' = p$

change of variable from $m$ to $r$

$\equiv \forall r : \text{prime nat} \ q = r \Rightarrow q' = \text{prime } n \land p' = p$

one-point $r$

$\equiv q' = \text{prime } n \land p' = p$

$\equiv q := \text{prime } n$

Using this transformer, $\text{check}$ is transformed to

$$ \forall m : q = \text{prime } m \Rightarrow \exists m' : q' = \text{prime } m' \land (p := \text{prime } m) $$

expand assignment

$\equiv \forall m : q = \text{prime } m \Rightarrow \exists m' : q' = \text{prime } m' \land m' = m \land p' = \text{prime } m$

one-point $m'$

$\equiv \forall m : q = \text{prime } m \Rightarrow q' = \text{prime } m \land p' = \text{prime } m$

change of variable from $m$ to $r$

$\equiv \forall r : \text{prime nat} \ q = r \Rightarrow q' = r \land p' = r$

one-point $r$

$\equiv q' = q \land p' = q$

$\equiv p := q$