459 (sparse array) An array A: [*[*rat]] is said to be sparse if many of its items are 0 . We can represent such an array compactly as a list of triples $[i; j; x]$ of all nonzero items $A\ i\ j = x \neq 0$. Using this idea, find a data transformer and transform the programs

After trying the question, scroll down to the solution.

§ We replace variable A with variable L with a domain such that L satisfies

L: [*[*nat*; *nat*; §r: *rat*· $r \neq 0$]] $\wedge\ \neg\exists n, m$: $\Box L$· $n \neq m \wedge L\ n\ 0 = L\ m\ 0\ \wedge\ L\ n\ 1 = L\ m\ 1$

Now the data transformer D is defined as

$\forall i, j$: *nat*· $\forall x$: *rat*· $(A\ i\ j = x \neq 0) = (\exists n$: $\Box L$· $L\ n = [i; j; x])$

Now to prove that this is a data transformer:

$\forall L$· $\exists A$· $\forall i, j$: *nat*· $\forall x$: *rat*· $(A\ i\ j = x \neq 0) = (\exists n$: $\Box L$· $L\ n = [i; j; x])$

$=$ UNFINISHED

(a) A:= [100*[100*0]]

§ SHOULD BE L:= [*nil*]

(b) x:= $A\ i\ j$

§ $\forall A$· $(\forall i, j$: *nat*· $\forall x$: *rat*· $(A\ i\ j = x \neq 0) = (\exists n$: $\Box L$· $L\ n = [i; j; x]))$

$\Rightarrow \exists A'$· $(\forall i, j$: *nat*· $\forall x$: *rat*· $(A'\ i\ j = x \neq 0) = (\exists n$: $\Box L'$· $L'n = [i; j; x]))$

$\wedge\ (x := A\ i\ j)$ expand assignment

$=$ $\forall A$· $(\forall i, j$: *nat*· $\forall x$: *rat*· $(A\ i\ j = x \neq 0) = (\exists n$: $\Box L$· $L\ n = [i; j; x]))$

$\Rightarrow \exists A'$· $(\forall i, j$: *nat*· $\forall x$: *rat*· $(A'\ i\ j = x \neq 0) = (\exists n$: $\Box L'$· $L'n = [i; j; x]))$

$\wedge\ x' = A\ i\ j\ \wedge\ A' = A$ one-point A'

$=$ $\forall A$· $(\forall i, j$: *nat*· $\forall x$: *rat*· $(A\ i\ j = x \neq 0) = (\exists n$: $\Box L$· $L\ n = [i; j; x]))$

$\Rightarrow$ $(\forall i, j$: *nat*· $\forall x$: *rat*· $(A\ i\ j = x \neq 0) = (\exists n$: $\Box L'$· $L'n = [i; j; x]))$

$\wedge\ x' = A\ i\ j$ UNFINISHED

$\Leftarrow$ $L' = L$

$\wedge$ **if** $\exists x$: *rat*· $\exists n$: 0,..#L· $L\ n = [i; j; x]$ **then** $\exists n$: $\Box L$· $L\ n = [i; j; x']$ **else** x'=0 **fi**

$\Leftarrow$ k:= 0. Q

where

Q

$=$ $L' = L$

$\wedge$ **if** $\exists x$: *rat*· $\exists n$: k,..#L· $L\ n = [i; j; x]$ **then** $\exists n$: k,..#L· $L\ n = [i; j; x']$ **else** x'=0 **fi**

$\Leftarrow$ **if** k=#L **then** x:= 0

else if $L\ k\ 0 = i\ \wedge\ L\ k\ 1 = j$ **then** x:= $L\ k\ 2$

else k:= k+1. Q **fi fi**

(c) A:= $(i;j) \rightarrow x \mid A$

SHOULD BE find n such that $L\ n\ 0 = i\ \wedge\ L\ n\ 1 = j$

if x=0 **then if** *found* **then** remove $L\ n$ **else** *ok* **fi**

else if *found* **then** replace $L\ n$ with $[i; j; x]$ **else** add $[i; j; x]$ **fi fi**