458 Let n be a natural constant. Let S: $n*nat$ be an implementer's variable. It is being reimplemented by R: $n*nat$ representing the same n naturals but in the reverse order.

(a) What is the data transformer?

(b) A user has variable i: nat and the operation

$$get \;=\; i:= S_i$$

Use your transformer from part (a) to transform get .

After trying the question, scroll down to the solution.

§ It is convenient to define the "reverse" operator $\leftarrow$. For any string S , define $\leftarrow S$ to be the reverse of S . Formally, if $n = \leftrightarrow S$,

$$\forall j\colon 0,..n \cdot S_j = (\leftarrow S)_{n-j-1}$$

and equivalently

$$\forall j\colon 0,..n \cdot (\leftarrow S)_j = S_{n-j-1}$$

and we can prove $\leftarrow$ is its own inverse

$$\leftarrow\leftarrow S = S$$

(a) What is the data transformer?

§ $D = R = \leftarrow S$

or equivalently

$D = S = \leftarrow R$

(b) A user has variable i: *nat* and the operation

$get = i := S_i$

Use your transformer from part (a) to transform *get* .

§ $\forall S \cdot D \Rightarrow \exists S' \cdot D' \land get$

$= \forall S \cdot S = \leftarrow R \Rightarrow \exists S' \cdot S' = \leftarrow R' \land (i := S_i)$ expand assignment in the old variable

$= \forall S \cdot S = \leftarrow R \Rightarrow \exists S' \cdot S' = \leftarrow R' \land i' = S_i \land S' = S$ one-point

$= \forall S \cdot S = \leftarrow R \Rightarrow i' = S_i \land \leftarrow R' = S$ one-point

$= i' = (\leftarrow R)_i \land \leftarrow R' = \leftarrow R$ $\leftarrow$ is self-inverse

$= i' = R_{n-i-1} \land R' = R$ contract assignment in the new variable

$= i := R_{n-i-1}$